

# Parameterizing DSDs and their variability



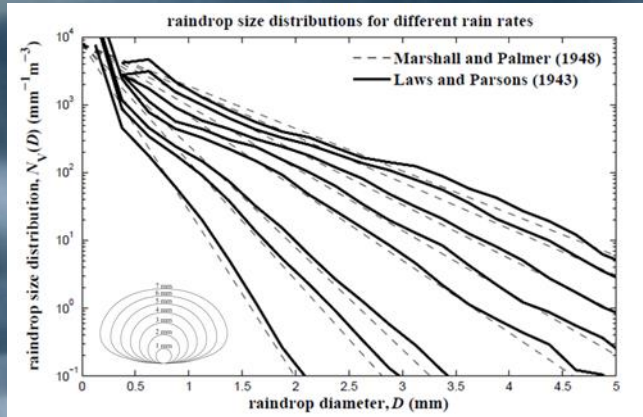
Remko Uijlenhoet

Ra2DW kick-off (5 Feb. 2024)

# Why raindrop size distributions?

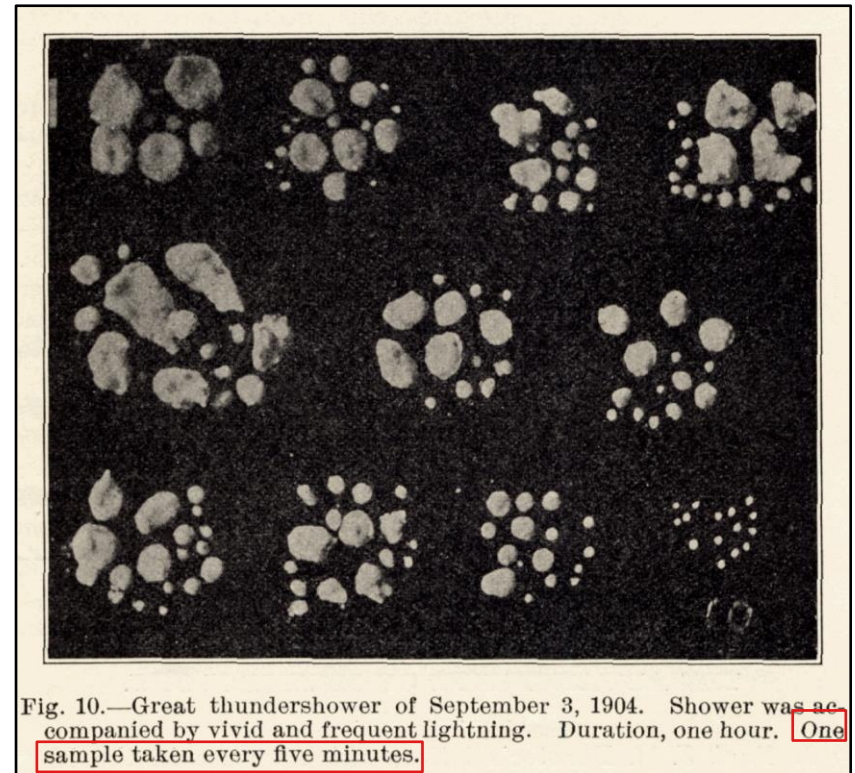
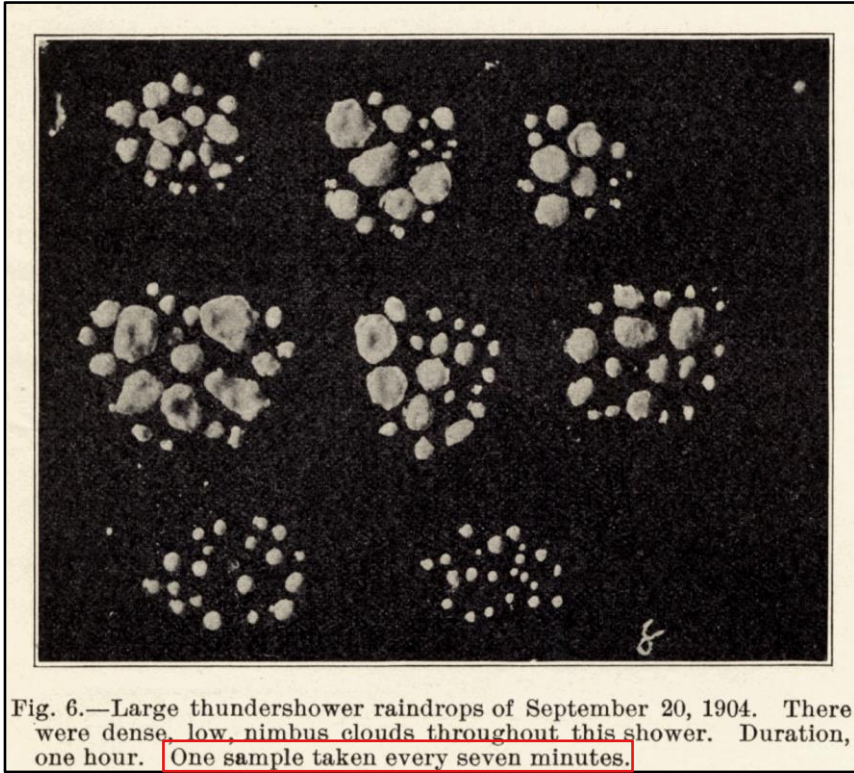


(Victoria Roberts, 2000)



(identim / Shutterstock)

# How to measure drop size distributions?



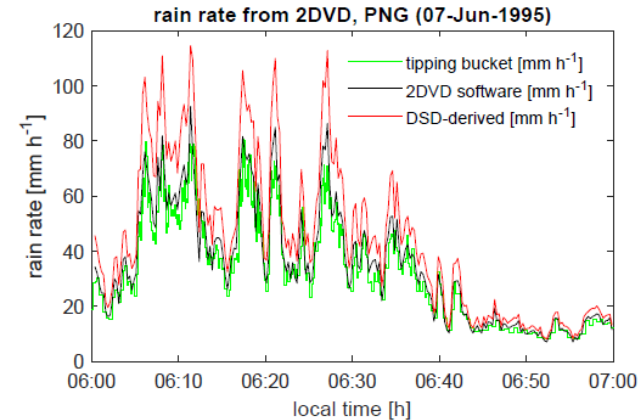
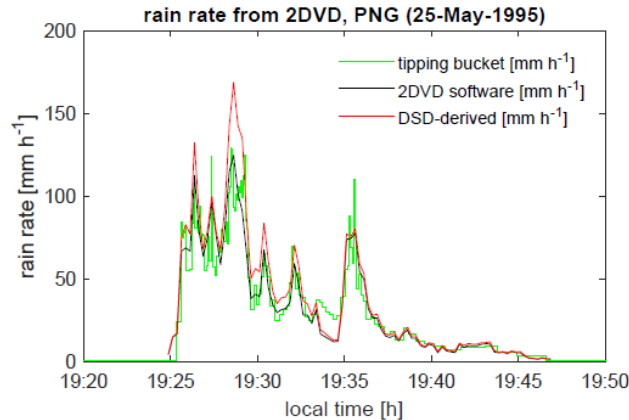
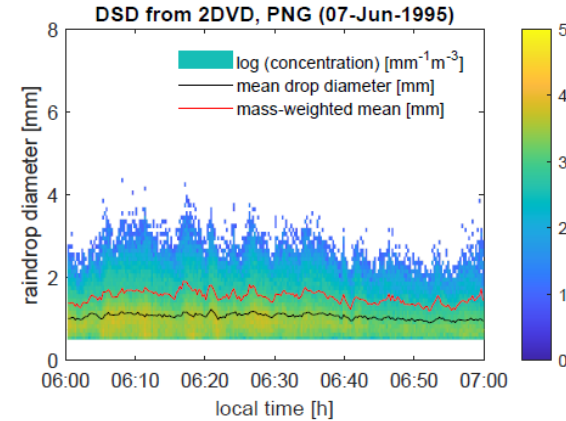
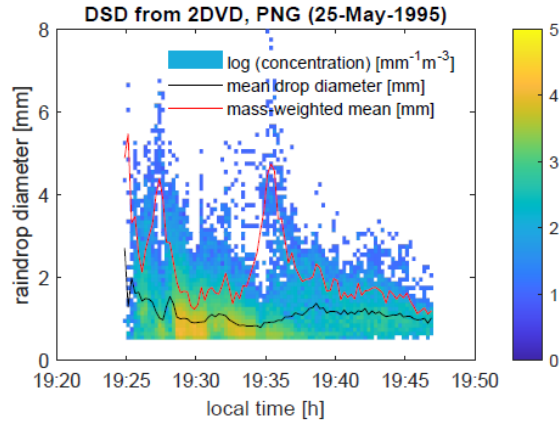
(W.A. Bentley, Studies of raindrops and raindrop phenomena, Monthly Weather Review, Oct. 1904)



# How to measure drop size distributions?



# Time series of drop size distributions



[2D-Video-Distrometer measurements in Lae / Papua New Guinea were carried out by JOANNEUM RESEARCH, Graz / Austria, under ESA Contract No. 9949/92/NL/PB(SC).]

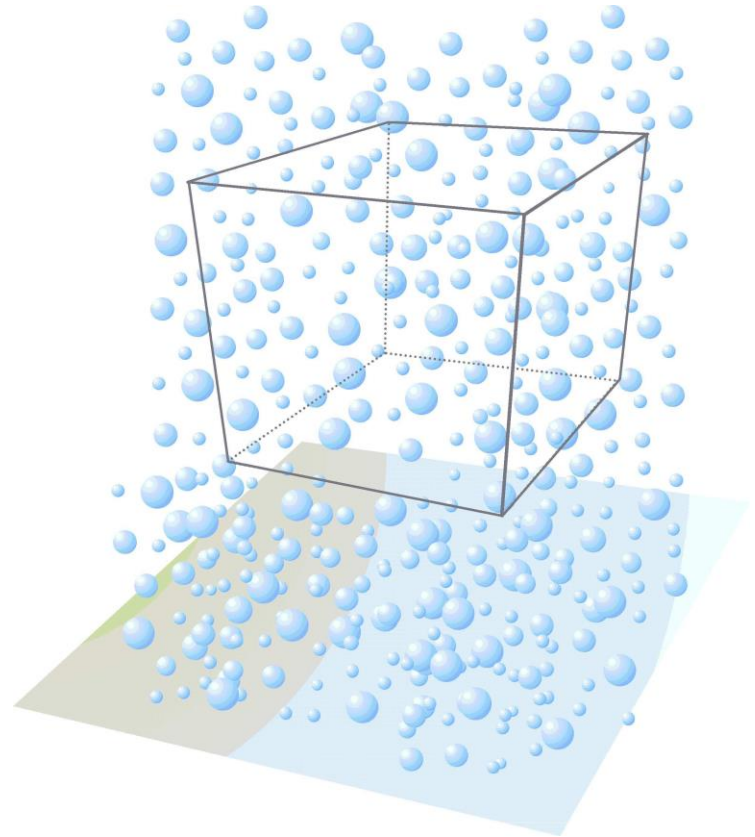
# What is a raindrop size distribution (DSD)?

$$n(d) = n_t f(d)$$

$$n(d) = n_0 p(x)$$

$$n_0 = n_t / \langle d \rangle$$

$$x = d / \langle d \rangle$$



(Courtesy of J.M. Porrà)

# How does rainfall rate depend on DSD?

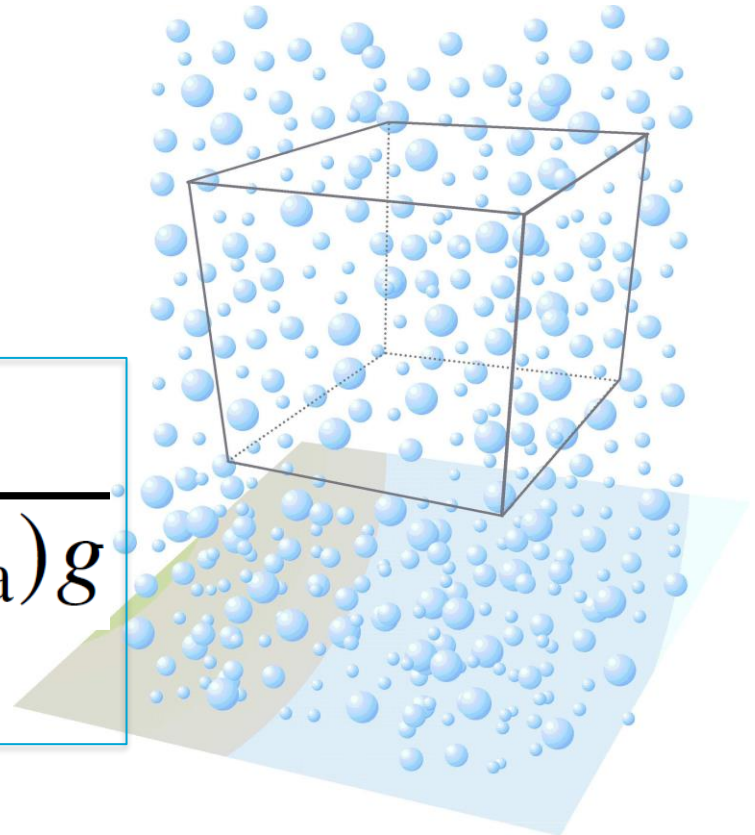
$$R = \int n(d) (\pi d^3 / 6) U dd$$

$$U = U_1 d^\gamma$$

$$\gamma = 1/2 \text{ and } U_1 = \sqrt{(\rho / \rho_a) g}$$

$$\gamma = 0.67 \text{ and } U_1 = 17.67$$

(Spilhaus, 1948; Atlas & Ulbrich, 1977)

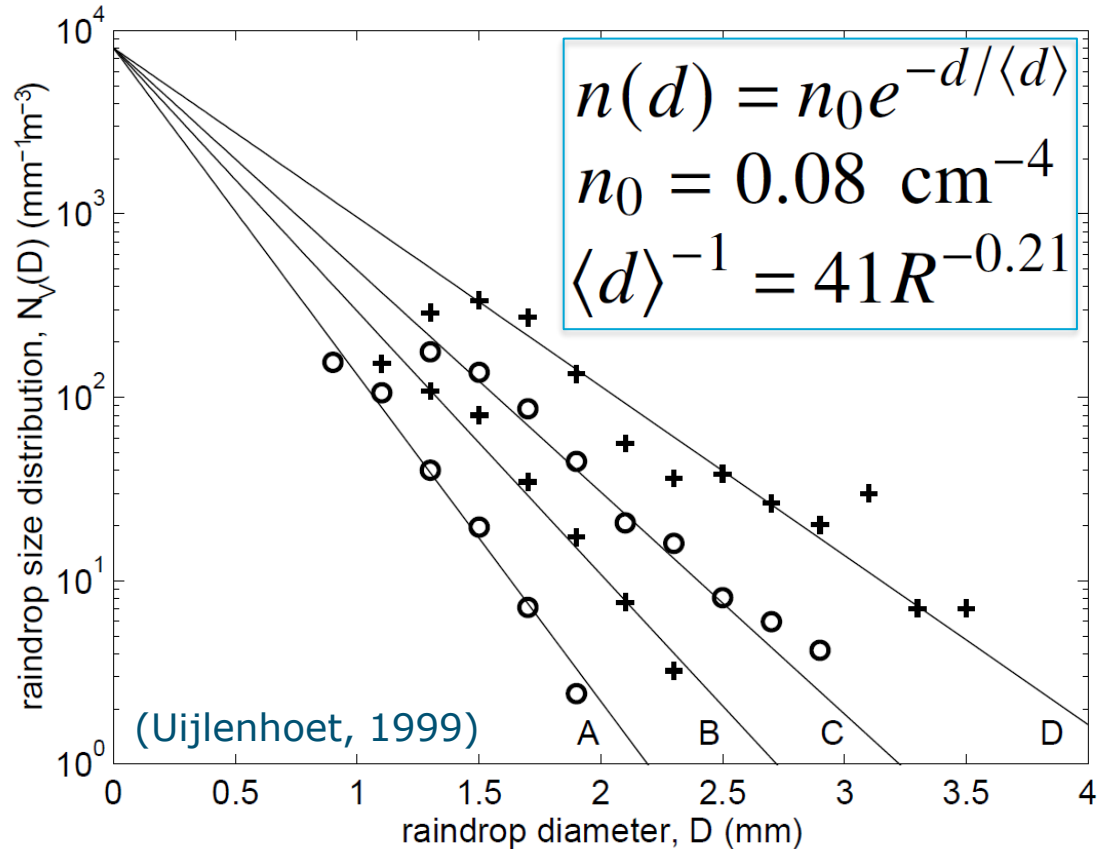


(Courtesy of J.M. Porrà)



# How does DSD depend on rainfall rate?

- Marshall and Palmer (1948)
- Family of drop size distributions
- “central limit theorem for drop size distributions”
- Most widely cited paper in radar meteorology





# Shapes of pdfs of normalized diameter

$$p(x) = e^{-x}$$

(Marshall & Palmer, 1948)

$$p(x) = \frac{(\mu + 1)^{\mu+1}}{\Gamma(\mu + 1)} x^{\mu} e^{-(\mu+1)x}$$

(Ulbrich, 1983)

$$p(x) = \frac{32}{3} x^{3/2} K_3(4\sqrt{x})$$

(Villiermaux & Bossa, 2009)

# Relaxing the assumption of a constant $n_0$

$$n(d) = n_t f(d)$$

$$n(d) = n_0 p(x)$$

$$n_0 = n_t / \langle d \rangle$$

$$x = d / \langle d \rangle$$

$$n_0 = \kappa R^\alpha$$

$$\langle d \rangle^{-1} = \lambda R^{-\beta}$$

$$\beta = (1 - \alpha) / (4 + \gamma)$$

# A scaling law for drop size distributions

1494

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## A General Formulation for Raindrop Size Distribution

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(Manuscript received 16 June 1993, in final form 11 March 1994)

$$n(d, R) = R^\alpha g(d/R^\beta)$$

$$M_r = \int d^r n(d) dd$$

$$M_r = a_r R^{b_r}$$

$$b_r = \alpha + (r + 1)\beta$$

$$\alpha + (4 + \gamma)\beta = 1$$

# Physical basis of Marshall-Palmer DSD?!

nature  
physics

ARTICLES

PUBLISHED ONLINE: 20 JULY 2009 | DOI: 10.1038/NPHYS1340

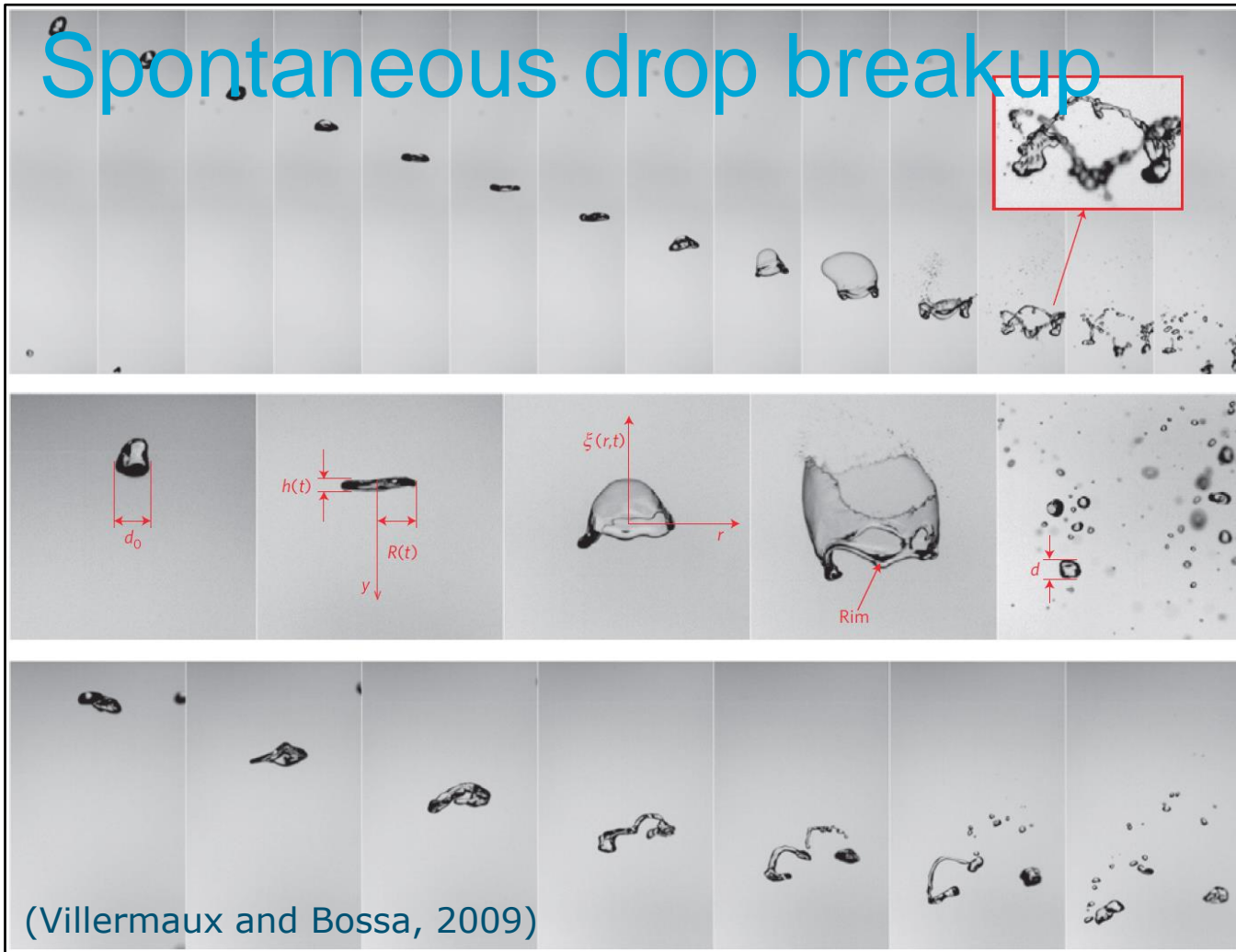
## Single-drop fragmentation determines size distribution of raindrops

Emmanuel Villermaux<sup>1,2</sup>★ and Benjamin Bossa<sup>1</sup>

Like many natural objects, raindrops are distributed in size. By extension of what is known to occur inside the clouds, where small droplets grow by accretion of vapour and coalescence, raindrops in the falling rain at the ground level are believed to result from a complex mutual interaction with their neighbours. We show that the raindrops' polydispersity, generically represented according to Marshall-Palmer's law (1948), is quantitatively understood from the fragmentation products of non-interacting, isolated drops. Both the shape of the drops' size distribution, and its parameters are related from first principles to the dynamics of a single drop deforming as it falls in air, ultimately breaking into a dispersion of smaller fragments containing the whole spectrum of sizes observed in rain. The topological change from a big drop into smaller stable fragments—the raindrops—is accomplished within a timescale much shorter than the typical collision time between the drops.



# Spontaneous drop breakup



(Villermaux and Bossa, 2009)



# Marshall & Palmer (1948), Ottawa

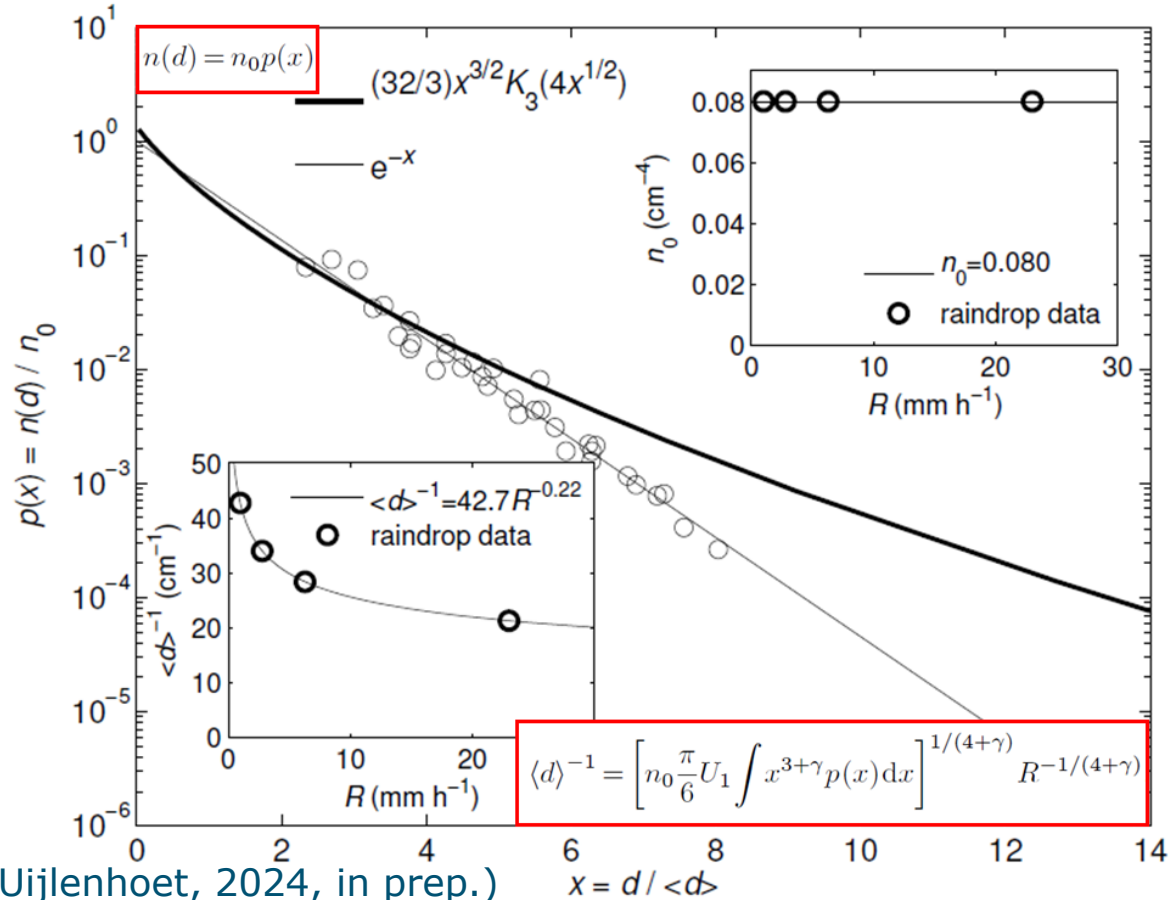
- $\alpha = 0$
- $\beta = 1/(4+\gamma)$
- largely size-controlled
- single-drop fragmentation?

nature physics ARTICLES  
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## Single-drop fragmentation determines size distribution of raindrops

Emmanuel Villermaux<sup>1,2\*</sup> and Benjamin Bossa<sup>1</sup>

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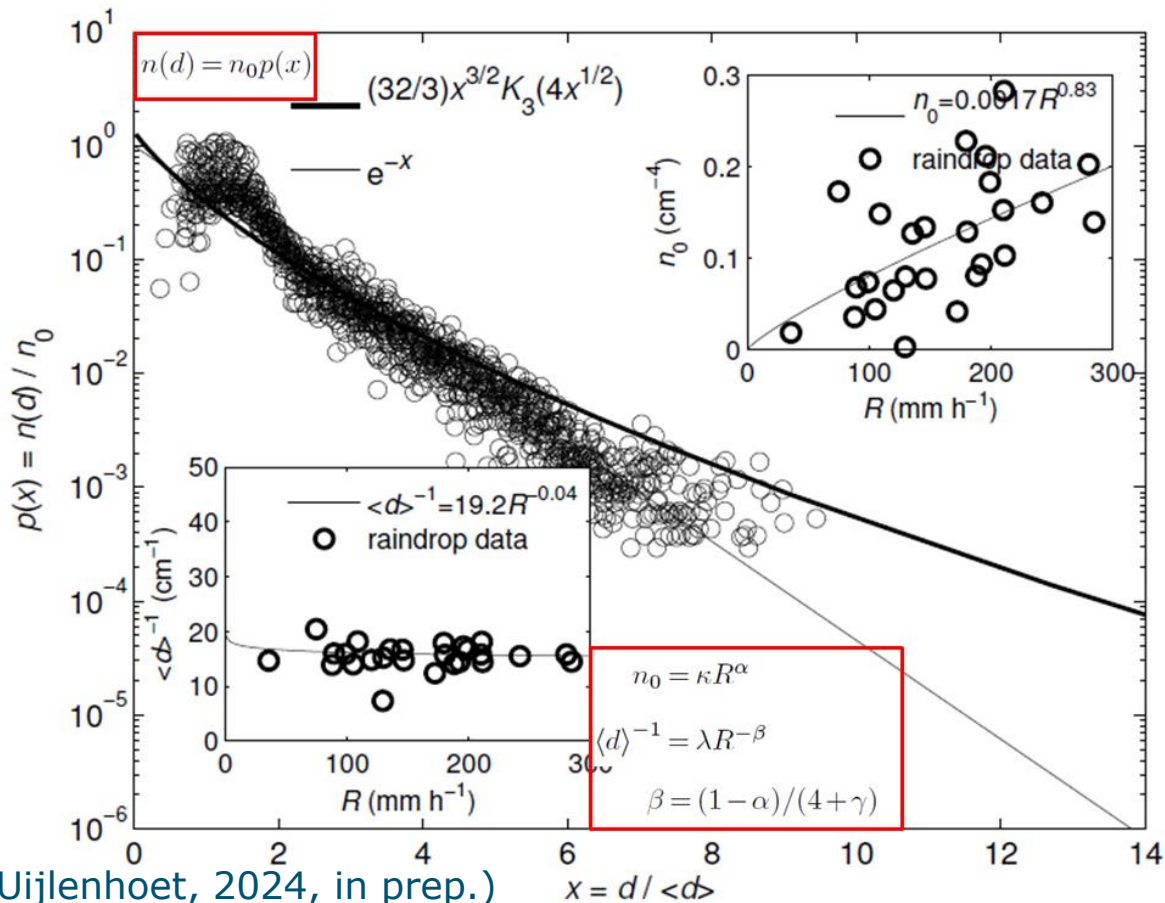
# Mueller (1962), Miami

- $\beta \approx 0$
- $\alpha \approx 1$
- largely number-controlled
- equilibrium rainfall

## Single-drop fragmentation determines size distribution of raindrops

Emmanuel Villermaux<sup>1,2\*</sup> and Benjamin Bossa<sup>1</sup>

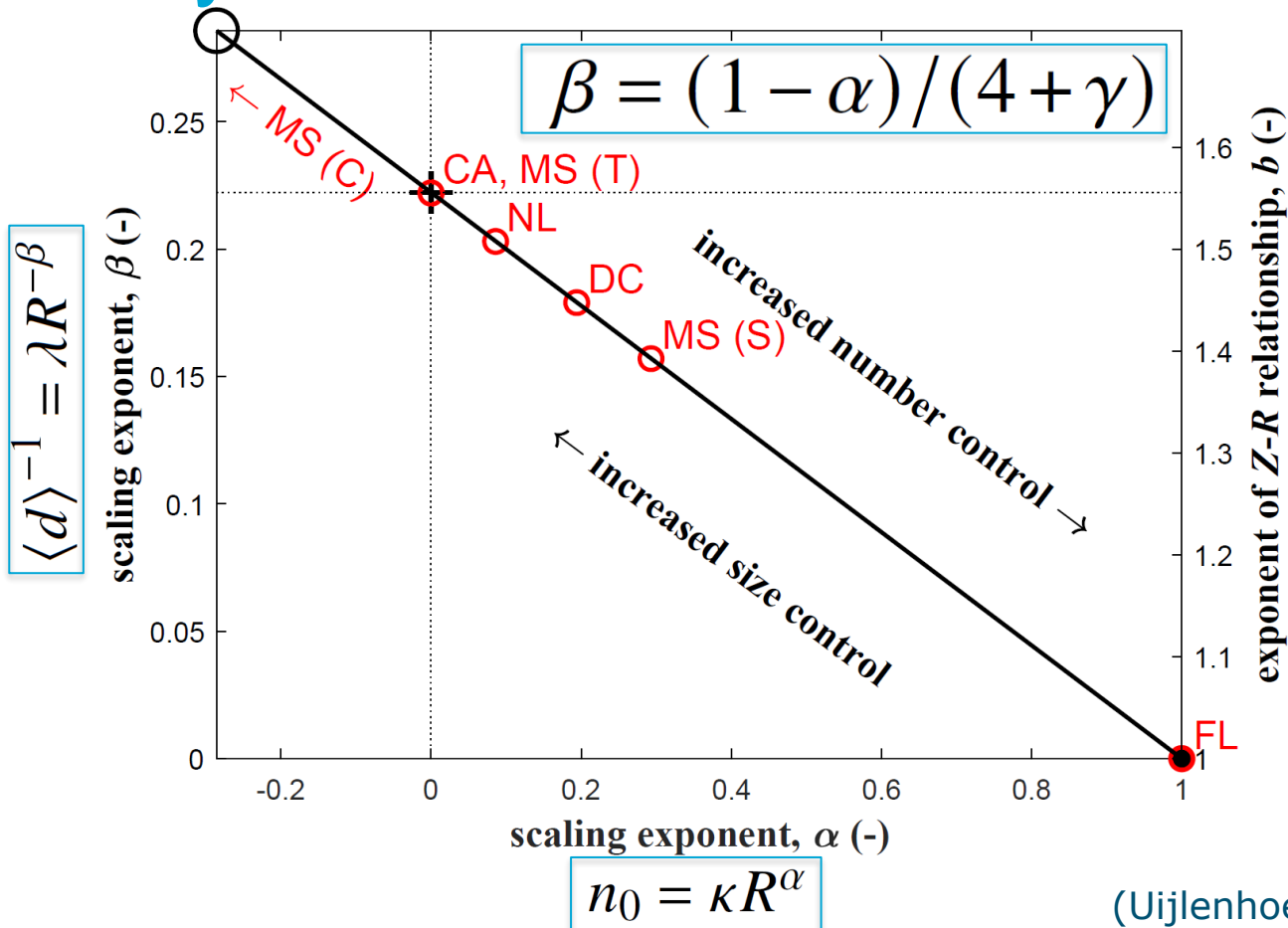
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(Uijlenhoet, 2024, in prep.)



# Summary



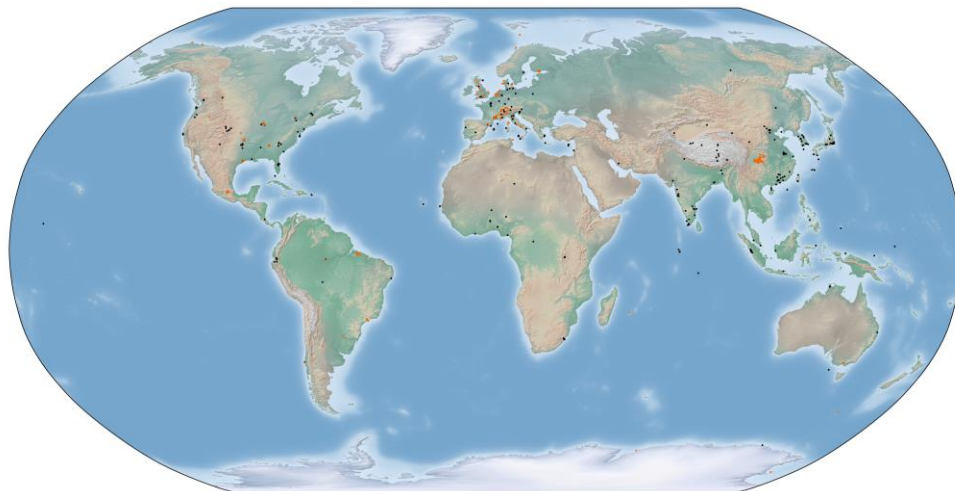
# Conclusions

The background of the slide features two tall, lattice-structured communication towers. The towers are silhouetted against a dramatic, overcast sky with dark, heavy clouds. The lighting is somewhat dim, suggesting an overcast day. The towers are positioned on the left and right sides of the frame, with the central area dominated by the text.

- Scaling law provides elegant framework for analysing spatio-temporal raindrop size distribution variability
- Confirmation of scaling law for a variety of climatic settings and for different types of disdrometric devices
- Exponential distribution provides good first guess for shape of scaled raindrop size distribution
- Spontaneous raindrop fragmentation scenario closely related to Marshall-Palmer rainfall
- Exponents of scaling law reflect controls on space-time variability of raindrop size distributions: from size-control to number-control

# DISDRODB initiative

- Towards a global data base of raindrop size distribution observations (Gionata Ghiggi et al.)



GET INVOLVED! Contribute with your data, algorithms, expertise and ideas.  
Contact us at [disdroidb@gmail.com](mailto:disdroidb@gmail.com)

# Thanks for your attention!



(Victoria Roberts, 2000)