Discrete and continuous multifractal cascade models: historical roots and application to turbulence

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Annual number of articles with « multifractal » in the title

source: Web of Science

began in 1985 and regular increase since then

total = 4246
Outline

• Historical remarks
• Two types of multifractal fields and how to analyze them
• Examples from the field of turbulence and other fields
• Conclusions, open questions and perspectives
Richardson (1922): energy cascade from large to small scales

Kolmogorov (1941), Obukhov (1941): dimensional analysis, leading to a scaling power spectra for velocity fluctuations in $k^{-5/3}$

The energy cascade in turbulence

Richardson and Kolmogorov: energy cascade
The energy cascade in turbulence

Experimental validation of the $-5/3$ law of Kolmogorov

$$E(k) \approx \varepsilon^{2/3} k^{-5/3}$$

Checked in many situations since the 1960s

Fig. 76 Normalized longitudinal velocity spectrum. x–Sandborn and Marshall; •–Grant, Stewart, and Moilliet; ©–Pond, Stewart, and Burling.
The energy cascade in turbulence

Intermittency

Batchelor and Townsend (1949): Experimental measurements of the dissipation: very strong fluctuations, are experimentally found, called « intermittency »

Obukhov (1962): locally averaged dissipation field, and assumption of lognormal fluctuations

Kolmogorov (1962): same hypothesis, and the variance depends on the log

\[ \varepsilon_r(x, t) = \frac{3}{4\pi r^3} \int_{|h| \leq r} \varepsilon(x + h, t) \, dh \]

averaged for a sphere of radius \( r \), and in assuming that for large \( L/r \) the logarithm of \( \varepsilon_r(x, t) \) has a normal distribution. It is natural to suppose that the variance of \( \log \varepsilon_r(x, t) \) is given by

\[ \sigma_r^2(x, t) = A(x, t) + 9k \log L/r, \quad (2) \]

Then the average is

\[ \varepsilon(M_1, M_2) = \frac{1}{4\pi r^3} \int_{V_{M_1, M_2}} \varepsilon \, dV. \]

* This assumption is not very restrictive as an approximate hypothesis since the distribution of any essentially positive characteristic can be represented by a logarithmically Gaussian distribution with correct values of the first two moments (see also Kolmogorov 1941b).
The energy cascade in turbulence

Spikes have a spatial structure

Intermittency of the dissipation, with power-law fluctuations

\[ B_{ee}(r) = \left( \frac{L}{r} \right)^\mu \]

Experimental results in Soviet Union (1963–1965): power-law correlation of the small scale dissipation

Gurvich and Zubkovskii (1963). Pond and Stewart (1965)
The energy cascade in turbulence

Yaglom’s discrete multiplicative cascade

Yaglom (1966) recursive multiplicative cascade model:
- the first discrete cascade model in turbulence
- multifractal properties

**Motivations**: Yaglom, as Kolmogorov’s student, wanted to build a model compatible with:
- Kolmogorov’s hypotheses
- long-range power-law correlations of epsilon as shown by experimental data

(i) generate lognormal statistics;
(ii) with power-law long-range correlations

A generic model for multiplicative cascades still today. Gives rise to multifractal statistics
The energy cascade in turbulence

Yaglom’s discrete multiplicative cascade: properties

\[ \epsilon(x) = \prod_{i=1}^{n} W_{i,x} \]

**Scaling**

\[ K(q) = \text{second characteristic function, or cumulant generating function, or scale invariant moment function} \]

\[ < \epsilon(x)^q > \approx \lambda^K(q) \]

\[ \lambda = \frac{L}{\ell} = 2^n \]

\[ K(q) = \log_2 < W^q > \]

**Power–law correlations**

\[ \mu = K(2) = \text{intermittency parameter} \]

\[ < \epsilon(x)\epsilon(x + r) > \approx r^{-\mu} \]

**Logarithmic relations for the generator**

\[ \gamma(x) = \log \epsilon(x) = \text{generator} \]

\[ < \gamma(x)\gamma(x + r) > \approx A - B \log r \]

\[ \sigma_v^2 = n\sigma_{\log W_i}^2 = \left( \frac{\sigma_{\log W_i}^2}{\log 2} \right) \log \left( \frac{L}{\ell} \right) = A' \log \left( \frac{L}{\ell} \right) \]

log law for the variance of \( \log \epsilon \), as originally assumed by Kolmogorov (1962)
A similar cascade model in two different fields

Geology, for metal ore deposition

Kriging (1951)

Obukhov (1962) Kolmogorov (1962)

Matheron (1962)
« regionalized variables »

de Wijs (1951, 1953)

Lognormal statistics

spatial average of a fluctuating quantity

\[ A_r(x) = \frac{1}{\text{vol}(B_r)} \int_{B_r(x)} e(x')dx' \]

Obukhov (1962)
Kolmogorov (1962)

Logarithmic relations for the generator

\[ \gamma(x) = \log e(x) \quad \text{generator} \quad \sigma_\gamma^2 = A' \log \left( \frac{L}{\ell} \right) \]

Kolmogorov (1962)
Yaglom (1966)

Discrete embedded multiplicative model

Yaglom (1966)

Turbulence, for the dissipation

de Wijs (1951, 1953)

Obukhov (1962)
Kolmogorov (1962)

See also:
Matheron’s theory of regionalized variables, Oxford University Press, 2019
Agterberg, Geomathematics, Springer, 2014
**Multifractal discrete cascades**

A discrete lognormal multifractal cascade
\[ N = 2^{16} = 65536 \]
\[ \mu = 0.2 \]
\[ < e > = 1 \]

\[ e(x) = \prod_{i=1}^{n} W_{i,x} \]

**Intermittency:** localized diverging pikes (singularities) with long-range correlations

**Discrete cascade models**

Early proposal in geosciences: De Wijs (1951, 1953)

Early lognormal proposal in turbulence: Yaglom (1966)

Black–and–white $\beta$–model: Novikov and Stewart (1964); Frisch et al. (1978)

Random $\beta$–model: Benzi et al. (1984)

$\alpha$–model: Schertzer and Lovejoy (1984)

$p$–model: Meneveau and Sreenivasan (1987)
Multifractal continuous cascades

Statistically, the cascade developed over a given scale ratio can be decomposed introducing any number of intermediary steps:

\[ X_{L \rightarrow \ell} = \prod_{i=1}^{n} X_{i} \]

Each \( X_{i} \) is independent and has the same law: called «Independent and identically distributed – iid»

\[ Y_{L \rightarrow \ell} = \sum_{i=1}^{n} Y_{i} \]

Each \( Y_{i} \) is independent and has the same law: called «Independent and identically distributed – iid»

«Infinitely divisible» law

log–Infinitely Divisible cascade models

For a infinitely divisible random variable \( Y \), for any integer \( n \), we can write:

\[ Y = \sum_{i=1}^{n} Y_{i} \]

where the \( Y_{i} \) are iid random variables

Each \( Y_{i} \) is independent and has the same law: called «Independent and identically distributed – iid»

Since Novikov (1994) it has been recognized that for a continuous cascade (in scale), i.e. a cascade that can be indefinitely densified, the log of the process belongs to ID distribution.

This means that **continuous cascades have log–ID distributions**.

Examples of models which are log–ID:

- lognormal model (Kolmogorov 1962)
- log–stable model (Schertzer and Lovejoy, 1987; Kida 1991)
- log–Gamma model (Saito, 1992)
- log–Poisson model (She and Leveque 1994, She and Waymire 1995, Dubrulle 1995)
### Multifractal framework in turbulence

**For the dissipation**

\[
\langle \epsilon^q \rangle \approx \ell^{-K(q)} \quad \text{scale invariance of moments} \\
K(q) \text{ moment function}
\]

\[\Pr(\epsilon > \ell^{-\gamma}) \approx \ell^{c(\gamma)} \quad \text{scale invariance of singularities pdf} \]

\[
\langle \epsilon^q \rangle = \int \ell^{-q\gamma} dp(\gamma) \approx \ell^{\min_{\gamma}\{c(\gamma) - q\gamma\}} \quad \text{Legendre transform relation between moment function and codimension function} \\
K(q) = \max_{\gamma}\{q\gamma - c(\gamma)\}
\]

**For the velocity**

\[
\langle |V(x + \ell) - V(x)|^q \rangle \approx \ell^{\zeta(q)} \quad \text{scale invariance of moments} \quad \zeta(q) \text{ moment function}
\]

\[\Pr(|V(x + \ell) - V(x)| > \ell^h) \approx \ell^{c(h)} \quad \text{scale invariance of singularities pdf} \quad c(h) \text{ codimension function}
\]

\[
\langle |V(x + \ell) - V(x)|^q \rangle = \int \ell^{qh} dp(h) \approx \ell^{\min_{h}\{c(h) + qh\}} \quad \text{Legendre transform relation between moment function and codimension function} \\
\zeta(q) = \min_{h}\{qh + c(h)\}
\]

Parisi and Frisch (1985) (and introduction of the word multifractal)
Outline

- Historical remarks
- **Two types of multifractal fields and how to analyze them**
- Examples from the field of turbulence and other fields
- Conclusions, open questions and perspectives
Two types of multifractals and two types of analysis

Singular measure; positive values with localized pikes
Directly produced by a multiplicative process
Ex: dissipation in turbulence

Scaling properties through coarse-graining: volume average of the small-scale singular field

\[ \epsilon_\ell(x) = \frac{1}{\text{vol}(B_\ell)} \int_{B_\ell(x)} \epsilon(x') dx' \]

\[ < \epsilon_\ell(x)^q > = C_q \ell^{-K(q)} \]

Stochastic processes with stationary increments (also called « multi-affine » or « non-stationary multifractals »)
Mixture of additive and multiplicative processes
Ex: velocity, passive scalars, in turbulence

Scaling properties through increments, or convolution with a kernel (wavelets) that suppresses a local trend

\[ < |X(x + \ell) - X(x)|^q > = C_q \ell^{\zeta(q)} \]
Two types of multifractals and two types of analysis

Singular measure; positive values with localized pikes
Directly produced by a multiplicative process
Ex: dissipation in turbulence

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Examples of analytical expressions:

- log-Poisson model: \( K(q) = c[(1 - \beta)q - 1 + \beta^q] \) (with \( c > 0 \) and \( 0 < \beta < 1 \))
- log-stable model: \( K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q) \) (with \( C_1 > 0 \) and \( 0 < \alpha \leq 2 \))
Two types of multifractals and two types of analysis

Stochastic processes with stationary increments (also called « multi-affine » or « non-stationary multifractals »)
Mixture of additive and multiplicative processes
Ex: velocity, passive scalars, in turbulence

Index of non-stationarity: $H = \zeta(1) \neq 0$

The curve $\zeta(q)$ is the sum of a linear trend and a nonlinear correction.

Examples:
- for the lognormal model, the correction is quadratic $\zeta(q) = qH - \frac{C_1}{\alpha - 1} (q^2 - q)$
- for the log-stable model, the correction is a power-law $\zeta(q) = qH - \frac{C_1}{\alpha - 1} (q^\alpha - q)$
- for the log-Poisson model, the correction is an exponential $\zeta(q) = qH - c[(1 - \beta)q - 1 + \beta^q]$

Scaling properties through increments, or convolution with a kernel (wavelets) that suppresses a local trend

$\langle |X(x + \ell) - X(x)|^q \rangle = C_q \ell^\zeta(q)$

$H = \zeta(1) \neq 0$

$\zeta(q) = qH - C_1 \left( \frac{q^2 - q}{\alpha - 1} \right)$

$\zeta(q) = qH - C_1 \left( q^\alpha - q \right)$

$\zeta(q) = qH - c \left( (1 - \beta)q - 1 + \beta^q \right)$
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Examples: multifractal measures

Dissipation in turbulence: Lagrangian data from a Direct Numerical Simulation

Examples: multifractal measures

Rainfall: many zeroes. Multifractal cascades with a fractal support

Examples: multifractal measures

Rainfall: a multiplicative model with zero values: a continuous $\beta$-multifractal model

Examples: non-stationary multifractal series

**Turbulence**: comparison between the velocity and passive scalars

Temperature field more intermittent than velocity field


Examples: non-stationary multifractal series

Climate: Greeland Ice-core

Multi-scaling from 0.4 to 40 kyr.

$H = 0.24 \pm 0.02$

Examples: non-stationary multifractal series

Fracture: scale invariance of crack surfaces

Examples: non-stationary multifractal series

Finance: exchange rates

Examples: non-stationary multifractal series

**Oceanology**: comparing temperature (passive scalar) with biologically active scalar (fluorescence, proxy of phytoplankton concentration)

\[
\frac{\partial \Theta}{\partial t} + u \cdot \nabla \Theta = D \nabla^2 \Theta + R(\Theta)
\]

transport  
biological activity


*Figure 3*. The empirical curves of scaling exponent structure functions \( \zeta(q) \) for temperature (thick continuous line), small-scale (dashed line) and large-scale fluorescence (thin continuous line) compared to the theoretical monofractal linear curve \( \zeta(q) = qH \) with \( H = 0.42 \) and \( H = 0.12 \) (discontinuous lines). The nonlinearity of the empirical curves indicates multifractality.
**Examples: non-stationary multifractal series**

**Wind energy**: multifractal properties of the wind power produced


Another method to extract scaling exponents: EMD-HSA method

Empirical Mode Decomposition + generalized Hilbert Spectral Analysis


Another method to extract scaling exponents: EMD-HSA method

Empirical Mode Decomposition + arbitrary order Hilbert Spectral Analysis

After decomposition the signal is written

\[ X(t) = \sum_{i=1}^{n} C_i(t) + r_n(t) \]

Hilbert transform of each mode

\[ C_i^H(t) = \int_{-\infty}^{+\infty} \frac{C_i(u)}{t-u} du \]

Construction of an “analytical” signal

\[ C_i^A(t) = C_i(t) + jC_i^H(t) = A_i(t)e^{j\theta_i(t)} \]

At each time step, extraction of a local amplitude \( A(t) \) and local frequency

\[ \omega_i(t) = \frac{d}{dt} \theta_i(t) \]

A time-frequency-amplitude analysis

Joint probability density function (frequency and amplitude)

\[ p(\omega, A) \]

Estimation of energy

\[ h(\omega) = \int_{0}^{\infty} A^2 p(\omega, A)dA \]

This is called Hilbert Spectral Analysis

Example, turbulence 5/3 spectrum:

Our contribution: arbitrary order HSA

\[ L_q(\omega) = \int_{0}^{\infty} A^q p(\omega, A)dA \]

\[ L_q(\omega) \sim \omega^{-\xi(q)} \]

\[ \xi(q) = 1 + \zeta(q) \]

Estimation of the multifractal moment function in the spectral space. Compares nicely with other methods (SF, wavelet leaders, DFA).

Less influenced by periodicities in the series.
Use of cumulants

Classical scaling methods:
• Estimate moments at different scale resolutions
• Display the scale invariance of the moments
• Estimate moments functions $\zeta(q)$ as slope of a fixed moment order, over a scale range
• Extract the parameters of a given multifractal model from the moment function $\zeta(q)$

Cumulant approach:
• Estimate the cumulant generating function at a given scale $\Psi(q) = \log < |V(x + \ell) - V(x)|^q >$
• Extract the parameters of the multifractal model at this scale
• Change the scale, and display the scale–dependence of the parameters

Advantages: a better precision for a given scale because the cumulant generating function is precisely estimated; can be used even when the scale invariance is not well verified: intermittent without perfect scaling

Delour et al., 2001; Eggers et al., 2001; Chevillard et al., 2005; Venugopal et al., 2006.
Use of cumulants

Cumulant generating function of $g_{\ell} = \log | V(x + \ell) - V(x) |$:
\[ \Psi(q) = \log < \exp(qg_{\ell}) > = \log < | V(x + \ell) - V(x) |^q > \]

This function is convex, as a second characteristic function, and can be developed using the cumulants:
\[ \Psi(q) = C_1 q + \frac{1}{2!} q^2 C_2 + \frac{1}{3!} q^3 C_3 + \ldots = \sum_{p=1}^{+\infty} \frac{q^p}{p!} C_p \]

The first cumulant is $C_1 = < g_{\ell} > = < \log | V(x + \ell) - V(x) | >$

For the log–stable model, the development is non–analytical and we have:
\[ \Psi(q) = C_1 q + C_\alpha q^\alpha \]

At scale $\ell$, $\alpha$ and $C_\alpha$ can be estimated precisely by plotting in log–log plot $\Psi(q) - C_1 q$ versus $q$. 

Evolution versus scale of the two parameters, for surf–zone oceanic velocity data
Large moments?

Extraction of figures from a paper (chosen randomly) using multifractal analysis of protein sequences

Fig. 3. Dimension spectra of measure $\mu$ from the CGRs of protein sequences of some organisms.

much too large order of positive moment

negative moments emphasize very small values... precision limitation of the data?
Divergence of moments of cascades

In the maths literature: « Gaussian multiplicative chaos »

\[
m(A) = \int_A e^{X(x) - (1/2)E[X(x)^2]} \, dx
\]

\[
e_\ell(x) = \frac{1}{\text{vol}(B_\ell)} \int_{B_\ell(x)} e(x') \, dx'
\]

\[
< (e_\ell)^q > = C\ell^{-K(q)}
\]

It is proven that moments are infinite if:

\[
< (e_\ell)^q > = \infty \iff K(q) \geq (q - 1)d
\]

This means a power-law tail of the pdf:

\[
p_{e_\ell}(x) \sim x^{-(q_D + 1)}
\]

and also for the survival function \( F(x) = \Pr(X \geq x) \sim x^{-q_D} \)

Hyperbolic law
Fat tail; heavy tail
Pareto law
Frechet law

Physics:
Mandelbrot, 1974
Scherzter et Lovejoy, 1987

Maths:
Peyrière et Kahane, 1976
Kahane, 1985
Guirvarc’h, 1987

Robert & Vargas, 2010
+ many recent works
Divergence of moments of cascades

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For moments larger than the threshold \( q_D \), experimental estimates are not infinite, but their value depend on the sampling

\[ K_e(q) = \begin{cases} 
K(q), & q \leq q_D \\
heq - \Delta_s, & q > q_D 
\end{cases} \]
Divergence of moments of cascades

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Schartzer and Lovejoy, 1992

Muzy et al, 2006
Divergence of moments of cascades

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\[
K_e(q) = \begin{cases} 
K(q), & q \leq q_D \\
heq - \Delta_s, & q > q_D 
\end{cases}
\]
Divergence of moments for velocity structure functions?

\[ \Delta V_\ell \propto (\varepsilon_\ell)^{1/3} \ell^{1/3} \]

\[ < (\Delta V_\ell)^{3q} > \sim < \varepsilon_\ell^{q} > \ell^{q} \]

Due to this K62 relationship, a divergence of the order \( \alpha \) for the dissipation corresponds to a divergence of order \( 3\alpha \) for velocity increments

\[ < (\Delta V_\ell)^q > = \infty \quad q \geq q_v = 3q_D \]

What is theoretically expected:

\[ \zeta_e(q) = \begin{cases} \zeta(q), & q \leq q_v = 3q_D \\ \Delta_s + \frac{q}{3}(1 - h_e), & q > q_v \end{cases} \]

Schertzer and Lovejoy, 1992
Muzy et al, 2006

Above the critical order of moments, the estimated value changes with the sampling size.
Divergence of moments for velocity structure functions?

\[ q_D \approx 2.4 \]

Dissipation

\[ q_V \approx 7 \]

Velocity increments


On October 7th 1994, a meeting was held between various European groups involved in experimental studies of 3D homogeneous turbulence. The aim of the meeting was to confront results obtained independently and see whether a general consensus on some properties of the velocity structure functions could be obtained. It turned out that agreement has been obtained on several characteristics of such functions, in particular on the values of scaling exponents (determined by using the technique described below), up to order 7. The participants thought that this fact was interesting to be reported. This does not mean that all the authors of the present letter agree on the significance of the result. In this letter, we essentially report facts and do not favour any particular interpretation.
Divergence of moments for velocity structure functions?

\[ q_D \approx 2.4 \]

Dissipation

\[ q_v \approx 7 \]

Velocity increments

\[ R_\lambda \sim 1000 \]

DNS data, divergence of order 2.4 for the dissipation

\[ R_\lambda \sim 1130 \]

DNS data, \( q_D \approx 8 \) for the velocity

Applying this criterion to our boundary layer data gives a maximum order of \( q=6 \), in full agreement with the previous qualitative analysis. Its application to a variety of data sets.


The DNS data at high \( Re \) suggest that the scaling exponents \( \zeta_p \) up to \( p = 8 \) are likely to be universal (insensitive to large-scale flow conditions), but it is unknown whether the universality applies at higher orders or to transverse exponents \( \zeta_{LT} \) or mixed exponents \( \zeta_{PQ} \).


Conclusions:

- Coarse-graining for measures.
- Structure functions, wavelets, EMD-HSA (or other methods) for non-stationary, or multi-affine fields.
- Different multifractal log-ID models exist, corresponding to different analytical expressions of the nonlinear part of the curves.
- Can be applied to many different fields.
- Limitations of the order of moment: sampling limitation, or divergence of moments. Several evidences for a divergence of moments of around 2.4 for the dissipation, and of $q_D = 7 \pm 1$ for velocity increments in turbulence.

Perspectives/what is still to be done?

- Turbulence: what relations between Navier-Stokes equations (deterministic) and the multifractal and intermittent properties of velocity and passive scalars?
- Predictions taking into account the long-range properties of scaling and multifractal fields.
- We know how to generate continuous multiplicative cascades. But how to generate a multi-affine stochastic process (in a « clean » and general way?
- Extensions to the multi-dimensional case.
STOCHASTIC ANALYSIS OF SCALING TIME SERIES
From Turbulence Theory to Applications

Cambridge University Press, 2016

TURBULENCE ET ÉCOLOGIE MARINE

François G. Schmitt
Ellipses, 2020