EGU NP Division Campfire: Scaling and Multifractals
February 1, 2022

A multifractal version of Navier-Stokes based model of velocity gradients in turbulence

Charles Meneveau (Johns Hopkins University)

Based on a sequence of contributions by:

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Laurent Chevillard (late 2000s, now ENS, Lyon, FR)

Michael Wilczek (mid 2010s, now Univ. Bayreuth DE)

Perry Johnson (mid 2010s, now UC Irvine, US)

Luo Hao (2021, Univ. Peking, CN)

Yi

Laurent

Michael

Perry

National Science Foundation
WHERE DISCOVERIES BEGIN

JOHNS HOPKINS UNIVERSITY

JOHNS HOPKINS Center for Environmental & Applied Fluid Mechanics

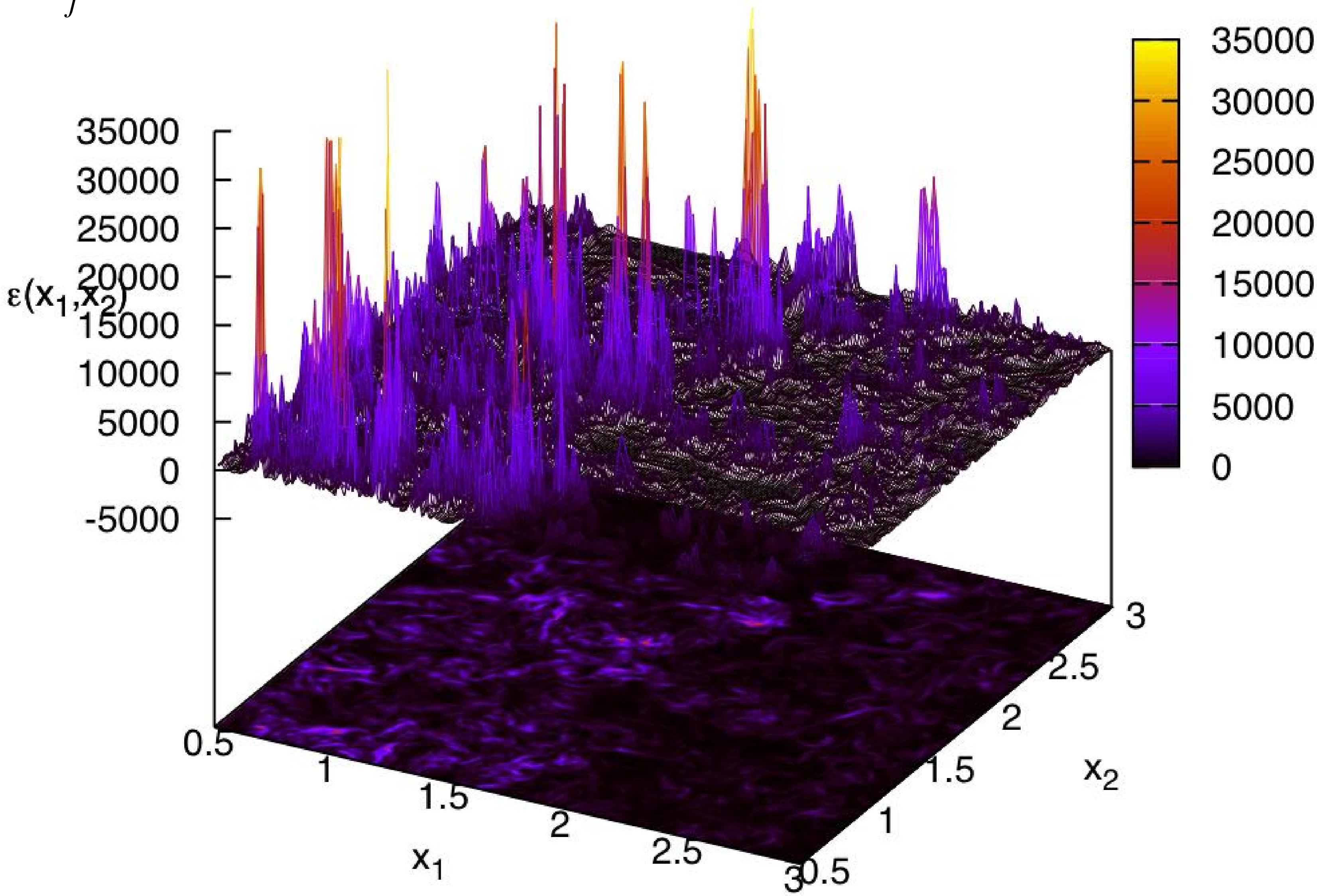
JHU Mechanical Engineering

idies Institute for Data Intensive Engineering and Science

Revisting the issue of small scale intermittency in regular 3D isotropic turbulence

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \varepsilon = 2\nu S_{ij} S_{ij}$$

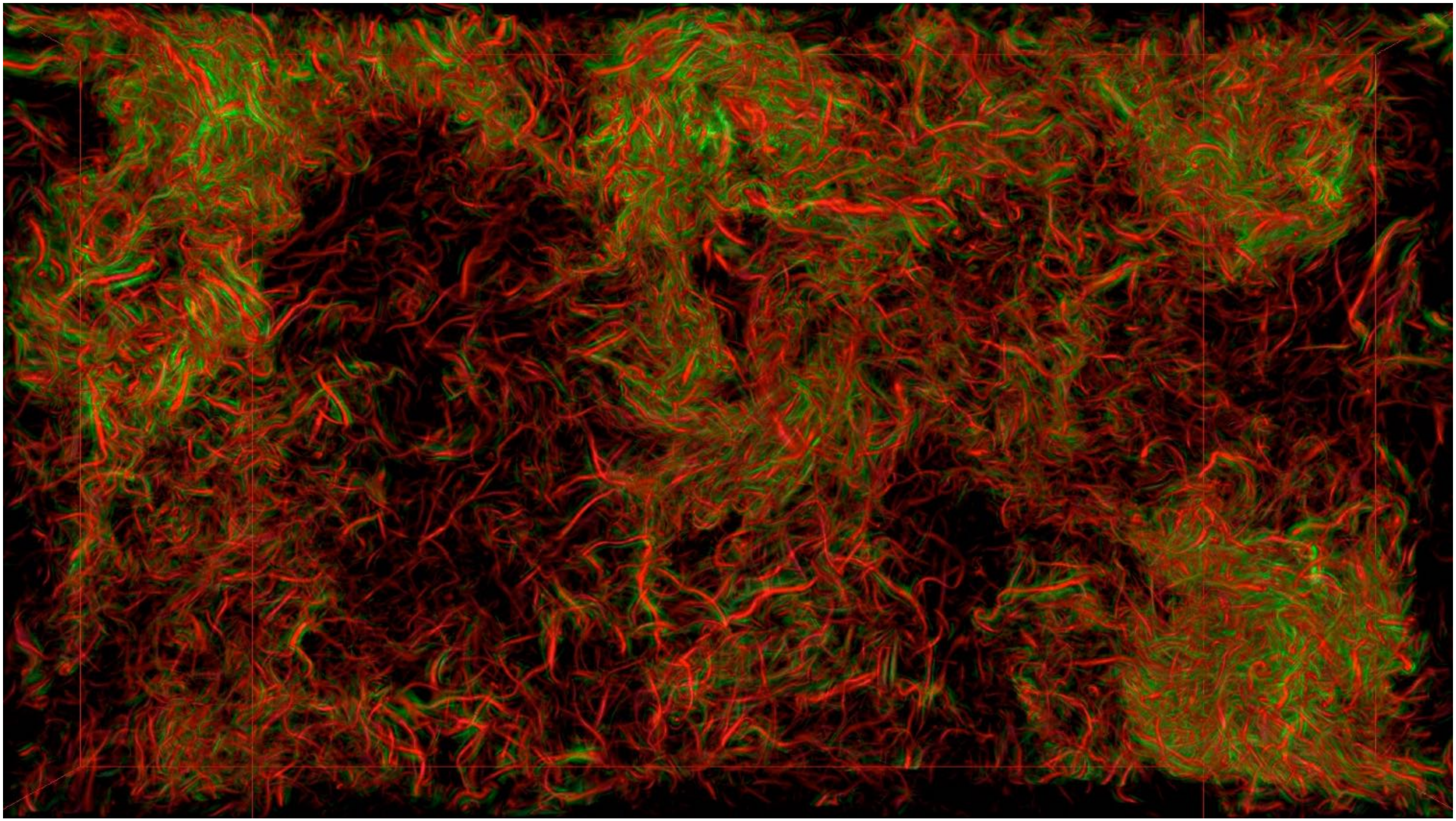
Rate of energy dissipation on a 512x512 plane in Re_λ=433, 1024^3 DNS of isotropic turbulence



Revisting the issue of small scale intermittency in regular 3D isotropic turbulence

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \omega^2 \propto \Omega_{ij}\Omega_{ij}$$

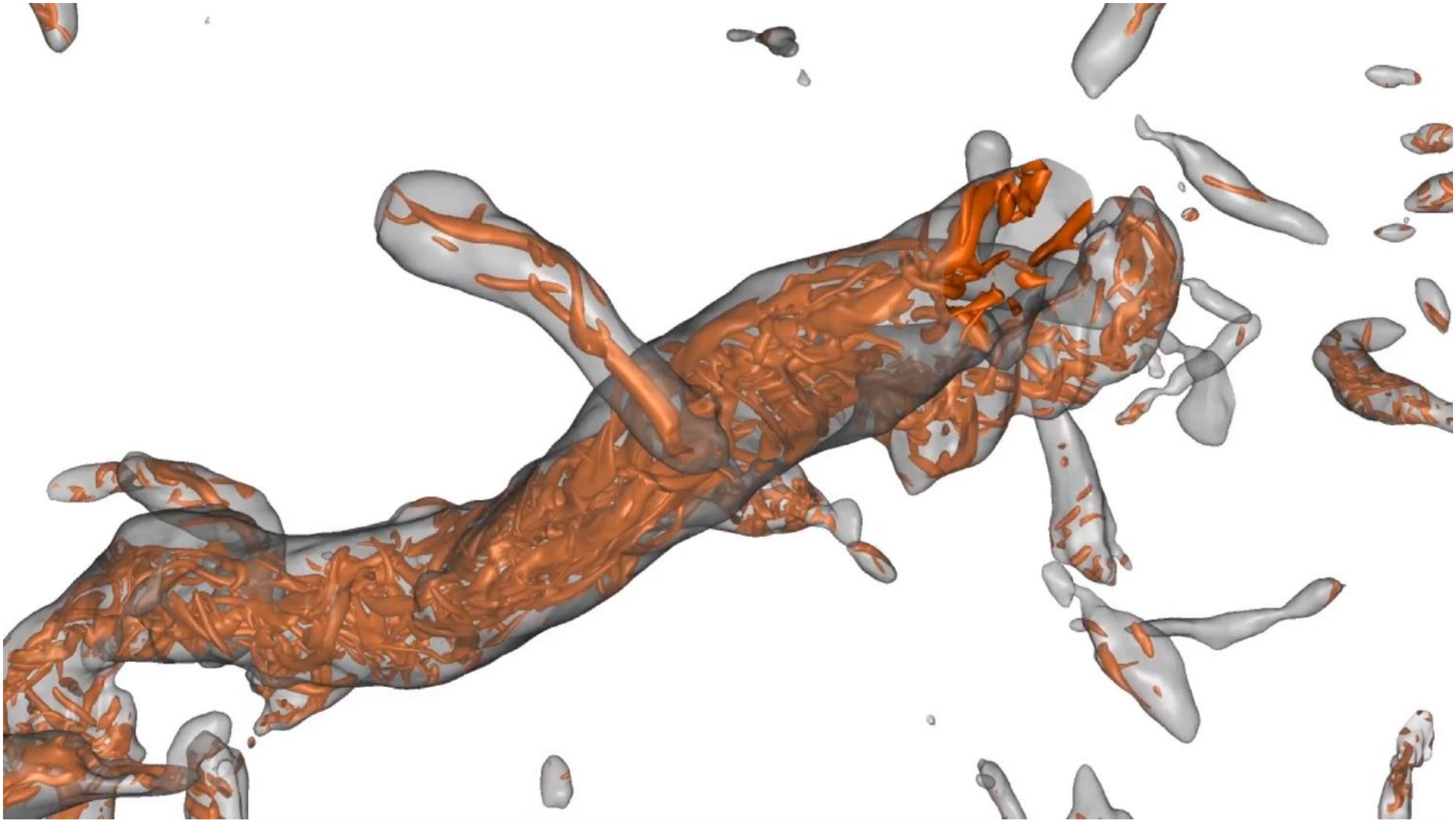
Enstrophy density plots in Re_λ=433
1024^3 DNS of isotropic turbulence



JHU database, Dr. Kai Buerger visualization

"Cascade" of vortices within vortices

2 & 3 scale vorticity iso-contours in Re_λ=433, 1024^3 DNS of isotropic turbulence (JHTDB)



Bürger, K., Treib, M., Westermann, R., Werner, S., Lalescu, C.C., Szalay, A., Meneveau, C. and Eyink, G.L., 2012. Vortices within vortices: hierarchical nature of vortex tubes in turbulence. arXiv preprint arXiv:1210.3325.

K41: assuming that mean dissipation tells entire story

$$\langle \varepsilon \rangle \sim \frac{u'^3}{\ell} \mathrm{Re}^0 \qquad \& \qquad \langle \varepsilon^{m/2} \rangle \sim \left(\frac{u'^3}{\ell} \right)^{m/2} \mathrm{Re}^0 \qquad \Rightarrow \left\langle \left| \frac{\partial u_1}{\partial x_1} \right|^m \right\rangle \Bigg/ \left\langle \left| \frac{\partial u_1}{\partial x_1} \right|^2 \right\rangle^{m/2} \sim \mathrm{Re}^0$$

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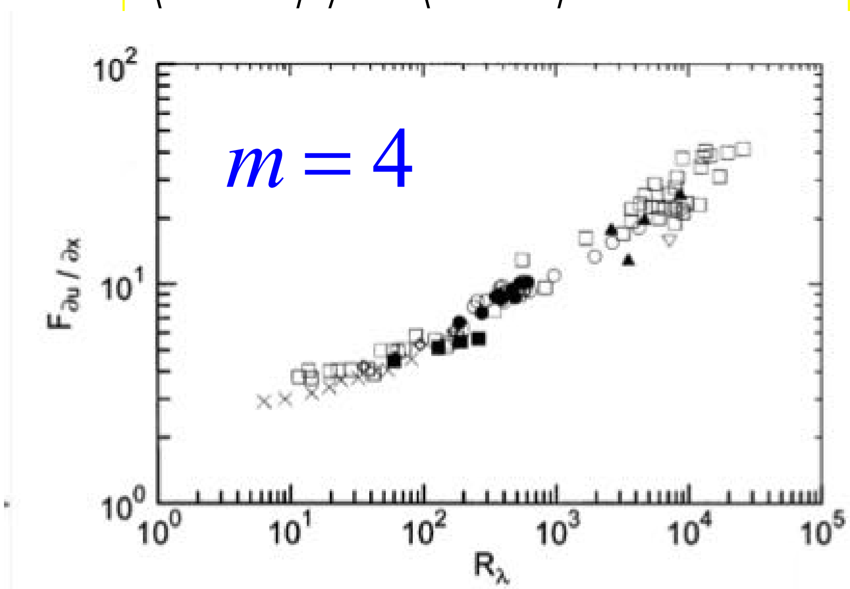
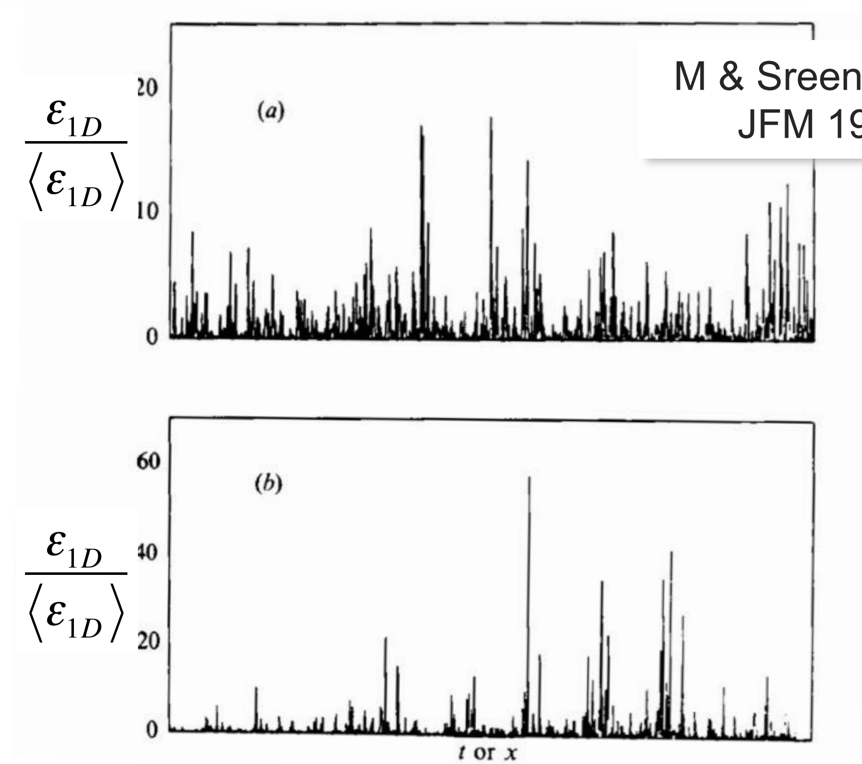
But: Intermittency in small-scale turbulence

$$\left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^m \right\rangle \Big/ \left\langle \left|\frac{\partial u_1}{\partial x_1}\right|^2 \right\rangle^{m/2} \sim \mathrm{Re}^{\alpha(m)}$$



Sreenivasan & Antonia ARFM 1997

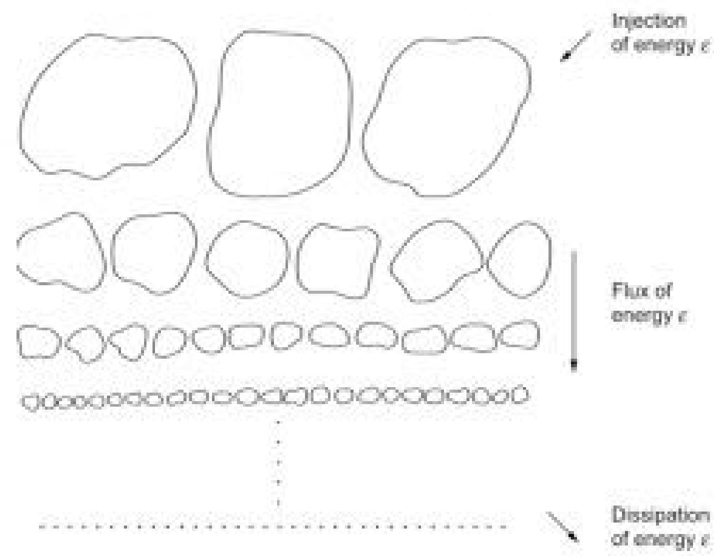


M & Sreenivasan
JFM 1991

Re ↑

Also: scaling of structure functions in inertial range
(Frisch 1995, see B. Dubrulle Campfire presentation)

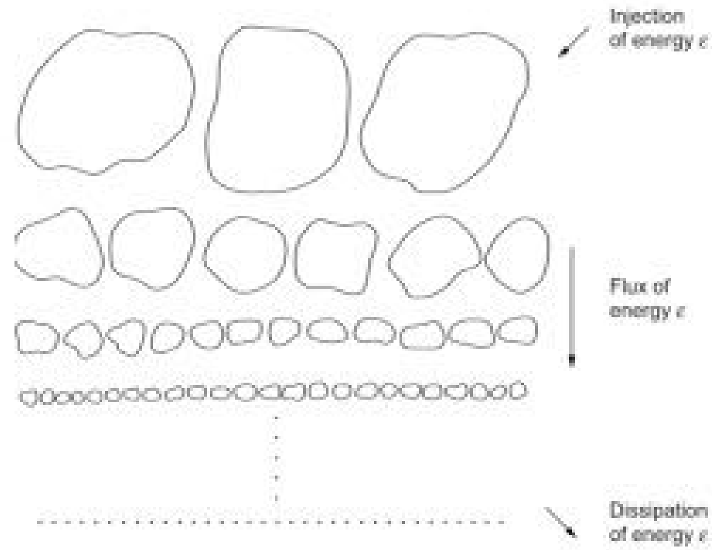
“Intermittency models”



(Frisch 1995, etc..)

- multifractals,
- She-Leveque,
- p-model,
- shell models
- ..
- Drawbacks: Limited descriptions of fluid mechanics, velocity increment, or scalars such as dissipation
- Little connection to Navier Stokes equations

"Intermittency models"



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- Drawbacks: Limited descriptions of fluid mechanics, velocity increment, or scalars such as dissipation
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- Here we take the view that reduced-order description still should be grounded in N-S, Lagrangian frame?
- Describe vectors and tensorial (geometric) structure

The velocity gradient tensor

Lagrangian evolution (CM, Annu Rev. Fluid Mech. 2011)

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

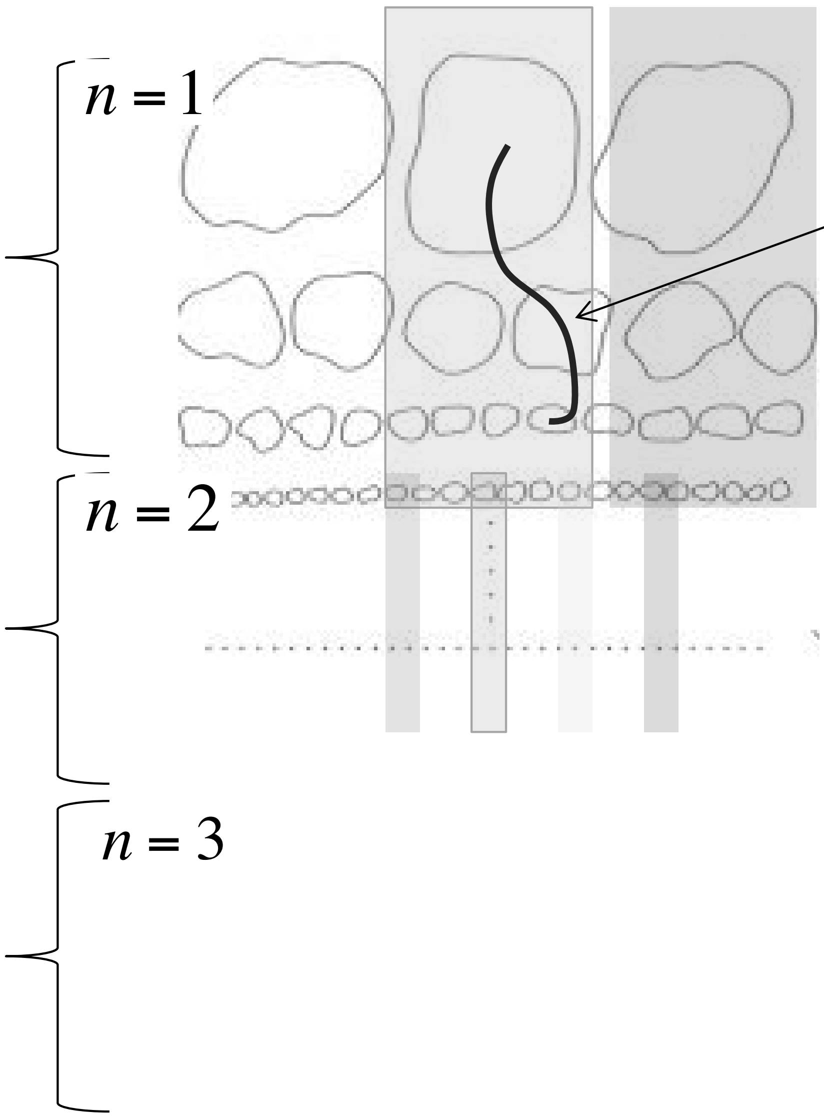
Summary: "Combined local and nonlocal cascade" in the Gradient Lagrangian Cascade Model of turbulence

$$\tau_1 = \beta^{N-1}\tau_K$$

$$\beta \sim 10$$

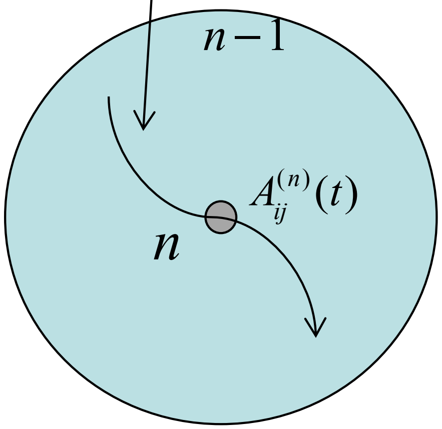
Nonlocal dynamics,

time-scales separation



Local dynamics from N-S

$$\frac{d\mathbf{A}}{dt} = -\mathbf{A}^2 + .. \mathbf{F}$$



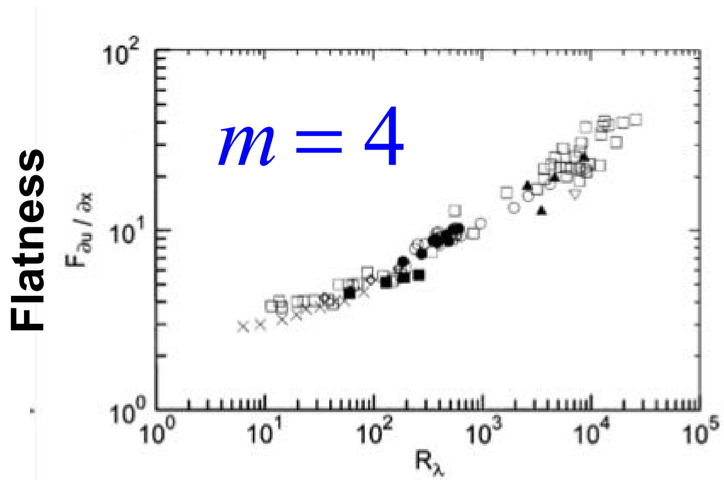
The velocity gradient tensor

Phenomenology (incompressible, NS):

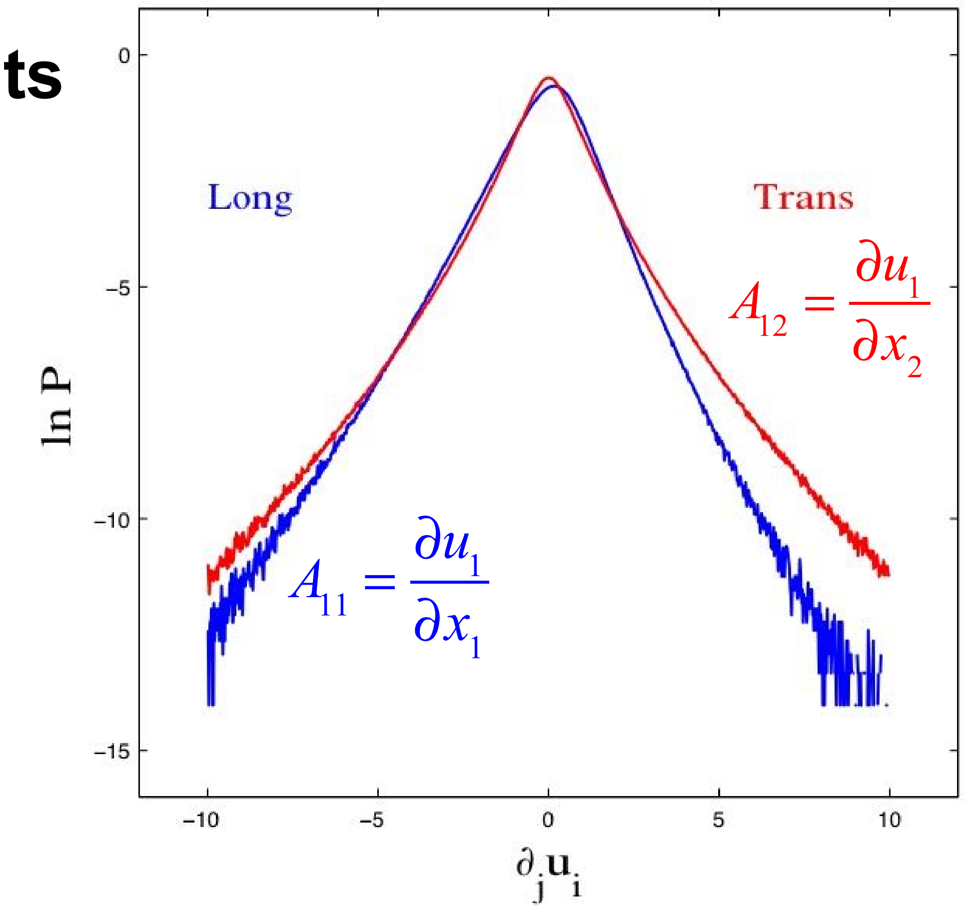
$$A_{ij} = \frac{\partial u_i}{\partial x_j}$$

intermittency: anomalous scaling of high-order moments

$$\frac{\left\langle \left| A_{11} \right|^m \right\rangle}{\left\langle \left| A_{11} \right|^2 \right\rangle^{m/2}} \sim \mathrm{Re}^{\alpha(m)}$$



intermittency: long tails in PDFs of gradients

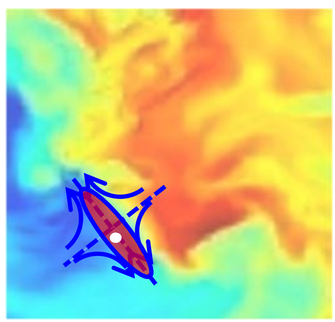


The velocity gradient tensor

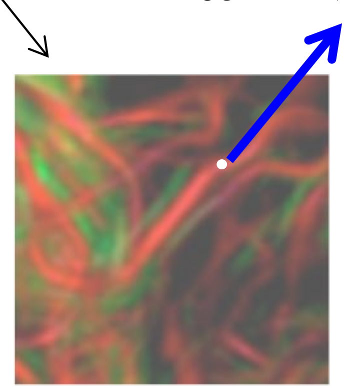
Phenomenology (incompressible, NS):

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

$$\omega = \nabla \times \mathbf{u}$$



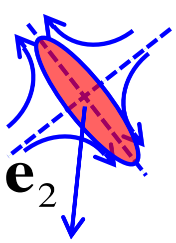
Strain-rate tensor:
eigen-values,
eigen-vectors



Rotation tensor
Vorticity vector

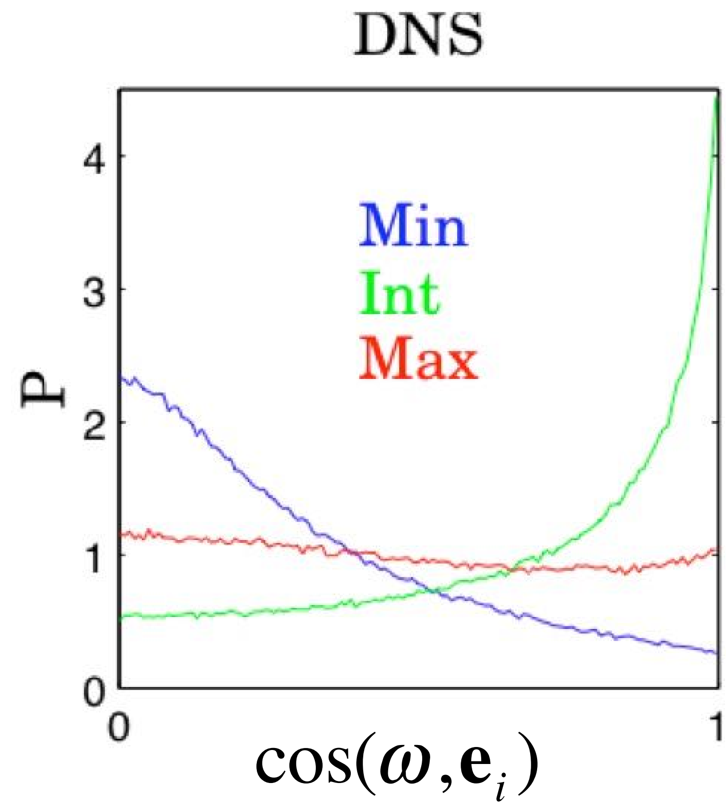
$\mathbf{e}_1 \quad (\lambda_1 > 0)$

$\mathbf{e}_3 \quad (\lambda_3 < 0)$



Geometric aspects:

E.g. preferential alignment of vorticity with intermediate strain-rate eigenvector (Ashurst et al. 1987, Kerr 1988, etc.):

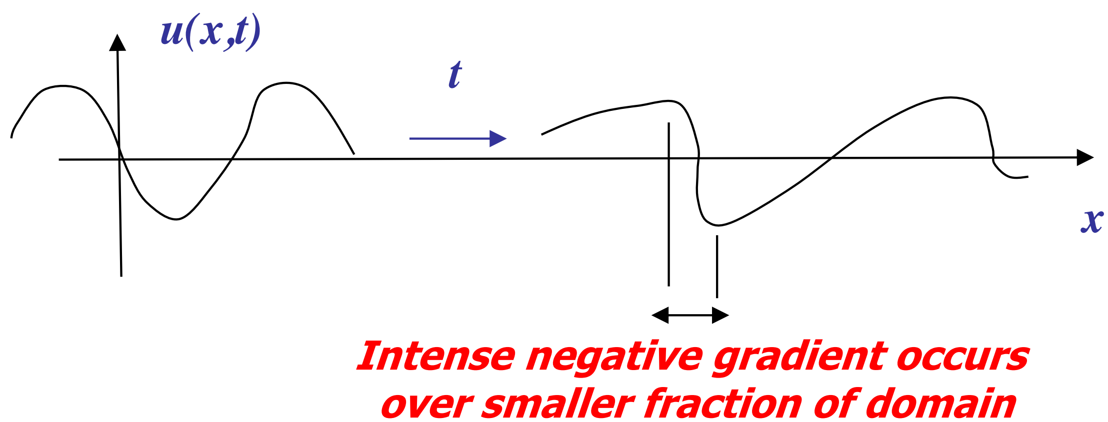


The velocity gradient tensor

in its Lagrangian evolution

Trivial case: 1-D inv Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$



$$A = \frac{\partial u}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + AA = 0$$

$$\frac{dA}{dt} = -A^2 \quad \Rightarrow A = \frac{1}{t-(-A_0^{-1})}$$

Finite time singularity
for negative initial
gradient points

The velocity gradient tensor: 3D

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij}$$

$$\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + g_i \right)$$

The velocity gradient tensor: 3D

$$A_{ij} = \frac{\partial u_i}{\partial x_j} = S_{ij} + \Omega_{ij} \qquad \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial t} + \frac{\partial u_k u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i + g_i \right)$$

$$\frac{dA_{ij}}{dt} = -\underbrace{\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right)}_{\text{Self-stretching}} \underbrace{- \left(\frac{\partial^2 p}{\partial x_i \partial x_j} - \frac{1}{3}\nabla^2 p \delta_{ij} \right) + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}}_{\text{unclosed}} + \underbrace{W_{ij}}_{\text{forcing}}$$

$$\frac{dA_{11}}{dt} = \ldots$$

$$\frac{dA_{12}}{dt} = \ldots$$

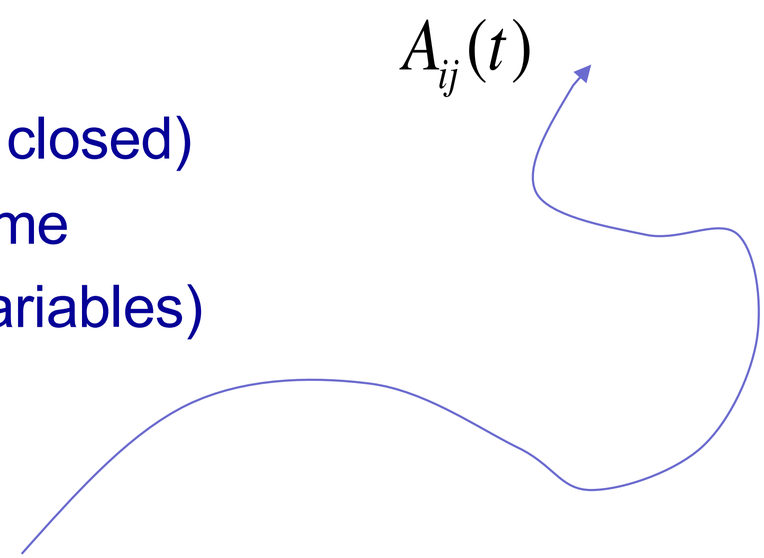
$$\frac{dA_{13}}{dt} = \ldots$$

...

...

$$\frac{dA_{33}}{dt} = \ldots$$

System of 9 (8) ODEs (not closed)
if viewed in Lagrangian frame
(dependent on non-local variables)



Families of models (review: Annu Rev Fluid Mech, 2011):

$$\frac{dA_{ij}}{dt} = -\left(A_{iq}A_{qj} - \frac{1}{3}A_{mn}A_{nm}\delta_{ij} \right) - \left(\frac{\partial^2 p}{\partial x_i \partial x_j} - \frac{1}{3}\nabla^2 p \delta_{ij} \right) + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q} + W_{ij}$$

- Restricted Euler Equation (1982 Viellefosse,1992 Cantwell..)
- Linear relaxation (Martin et al.. 1998)
- Specified lognormal and stochastic (Chen, Pope, Girimaji)
- Tetrads.. (Chertkov, Pumir)
- Recent Fluid Deformation closure (Chevillard & CM, 2006)
- Linear combination, with a shell-model approach (Biferale et al.)
- Gaussian and “enhanced” Gaussian (Wilczek & CM, JFM 2014)
- Recently Deformed Gaussian Fields Model

  (Johnson & CM, JFM 2017)

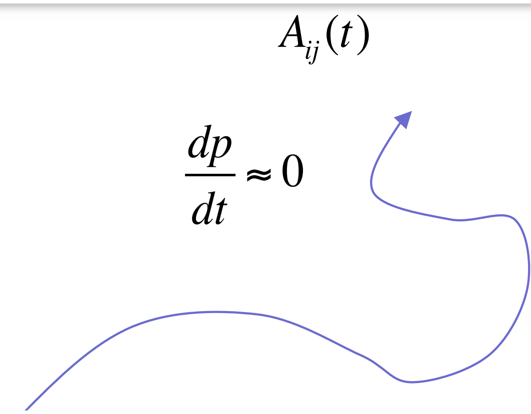
Pressure Hessian model: assume pressure is slowly varying along Lagrangian trajectories over short time-period τ

$$\frac{dp}{dt} \approx 0 \rightarrow \quad \nabla\nabla p : \frac{\partial^2 p}{\partial x_i \partial x_j}$$

$$\nabla\nabla\left(\frac{dp}{dt}\right) = \frac{d\nabla\nabla p}{dt} + \nabla\mathbf{u}\nabla\nabla p + \nabla\mathbf{u}^T\nabla\nabla p$$

$$\rightarrow \frac{d\nabla\nabla p}{dt} \approx -\nabla\mathbf{u}\ \nabla\nabla p - \nabla\mathbf{u}^T\ \nabla\nabla p$$

$$\rightarrow \nabla\nabla p(t) \approx e^{-\tau\nabla\mathbf{u}} \cdot \nabla\nabla p(0) \cdot e^{-\tau\nabla\mathbf{u}^T}$$



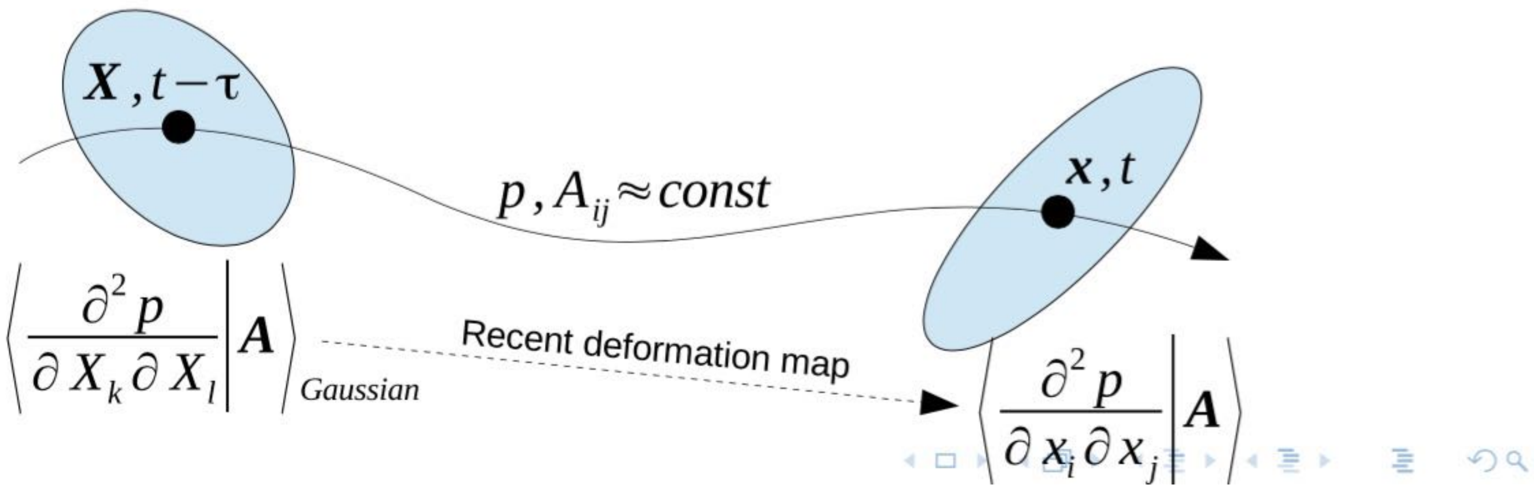
Olroyd (upper convective) derivative = zero

Chevillard & M (2006): Recent Fluid Deformation Map

C&M 2006 assumed I.C. is an isotropic tensor (Lagrangian restricted Euler)

$$\left\langle \frac{\partial^2 p}{\partial X_i \partial X_j} \middle| \mathbf{A} \right\rangle \approx \frac{1}{3} \left\langle \frac{\partial^2 p}{\partial X_k \partial X_k} \middle| \mathbf{A} \right\rangle \delta_{ij}.$$

The Recent Deformation of Gaussian Fields (RDGF) closure
(P. Johnson & CM, JFM 2016)



$$\nabla\nabla p(t) = P_{ij}(t) \approx \exp(-\tau \mathbf{A}) \cdot \nabla\nabla p(0) \cdot \exp(-\tau \mathbf{A}^{\mathrm{T}})$$

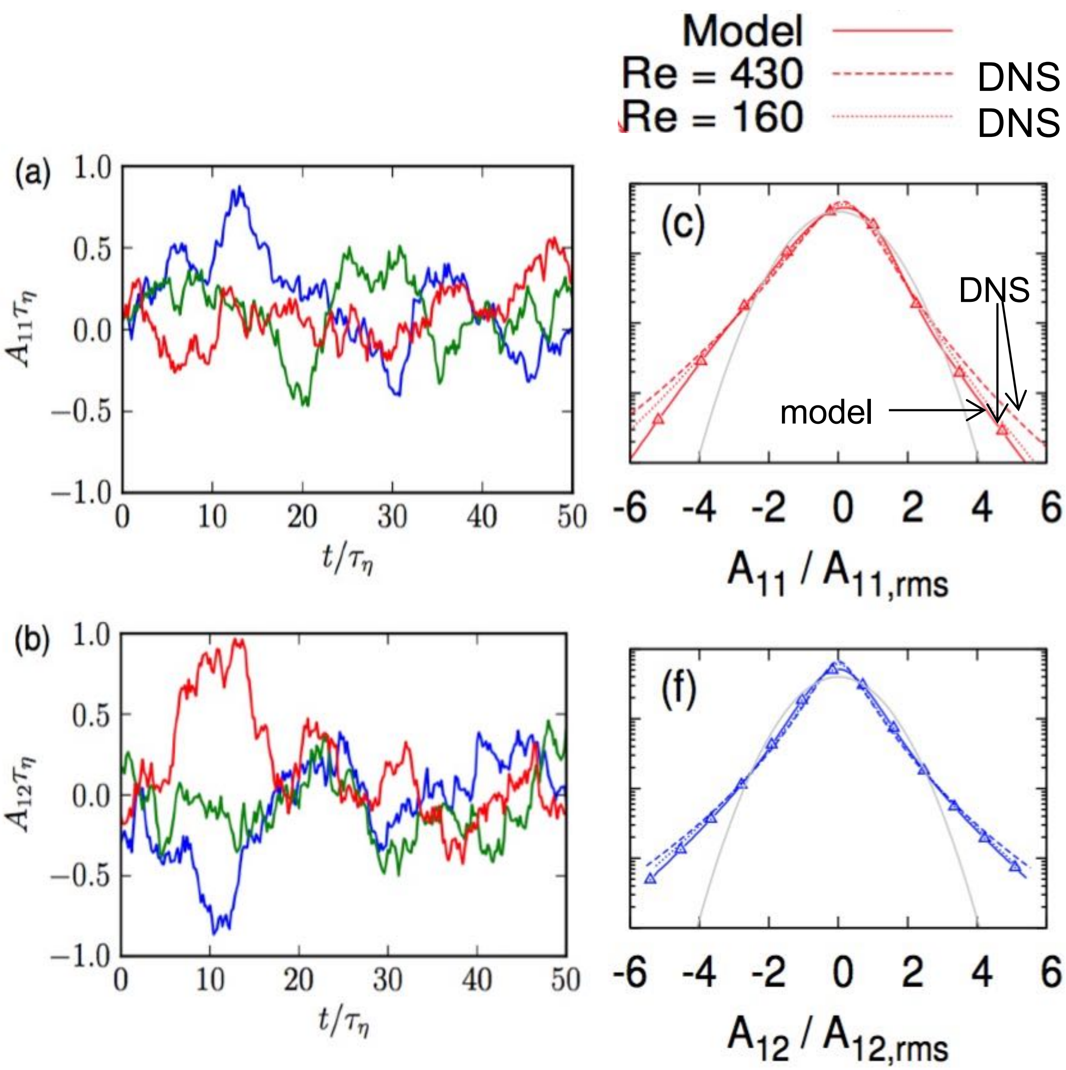
$$\langle \widetilde{\boldsymbol{H}}(\boldsymbol{x}_1) | \boldsymbol{\mathcal{A}}_1 \rangle = \alpha \left(\boldsymbol{\mathcal{S}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{S}}_1^2\right) \boldsymbol{I} \right) + \beta \left(\boldsymbol{\mathcal{W}}_1^2 - \tfrac{1}{3}\mathrm{Tr}\left(\boldsymbol{\mathcal{W}}_1^2\right) \boldsymbol{I} \right) + \gamma \left(\boldsymbol{\mathcal{S}}_1 \boldsymbol{\mathcal{W}}_1 - \boldsymbol{\mathcal{W}}_1 \boldsymbol{\mathcal{S}}_1 \right)$$

Gaussian closure for conditional pressure Hessian

(Wilczek & M, JFM, 2015)

The Recently-Deformed Gaussian Fields (RDGF) Model

$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3} A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \boldsymbol{\tau}_K) \right] dt + dF_{ij}(\boldsymbol{\tau}_K)$$



3 scalar parameters, fixed by 3 kinematic conditions

Results:

- ✓ Skewness
- ✓ Alignments
- ✓ Joint PDF (Q,R)

However:

- ➢ Behavior like at a **fixed Re** (Re_λ ~ 60 -120).

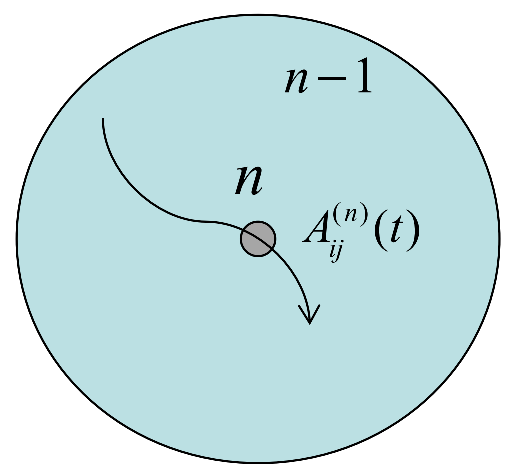
- ❖ Can't describe continued "widening" of tails in PDF at large Re

A multiple time-scale variant (Johnson & M Phys.Rev. Fluids 2017):

Instead constant $\tau_K = \sqrt{\nu / \langle \varepsilon \rangle}$, use "variable" background $\tau_K(t) = \sqrt{\nu / \varepsilon(t)}$

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n)$$

$$\tau_n(t) = \frac{1}{\beta} \left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)} \right)^{-1/2}, \quad n = 2,3,\ldots\ N$$



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$$\boldsymbol{\tau}_1 = \beta^{N-1} \boldsymbol{\tau}_K$$

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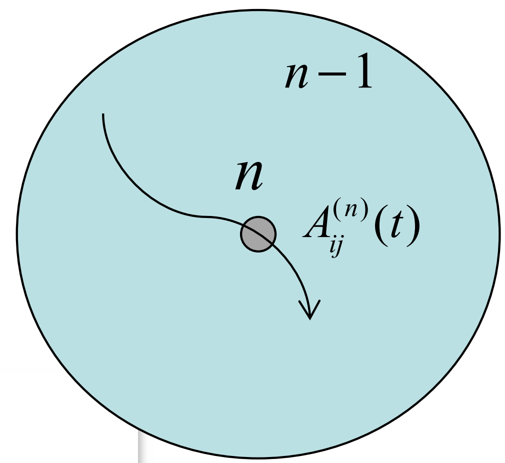

- Free parameter: *β=10* (fitted)
- levels: scale-separation needed in model
- not a "shell cascade model" (scale ratio = 2)

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local-in-scale interactions: from N-S

non-local interactions

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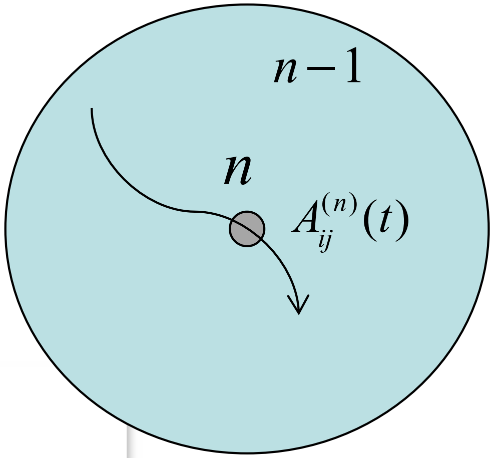

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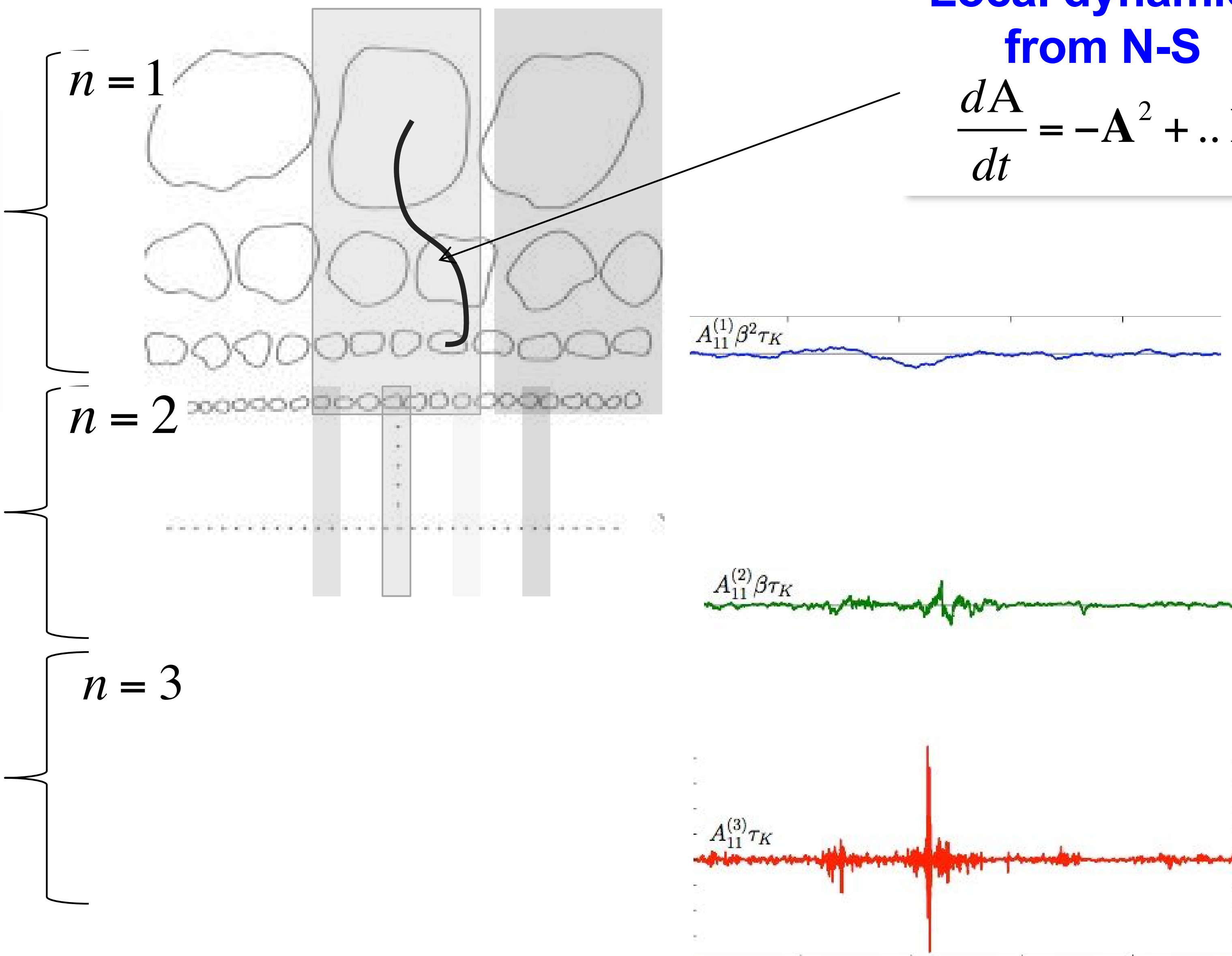
"Combined local and nonlocal cascade" in the Gradient Lagrangian Model of turbulence

Nonlocal dynamics, modulated time-scales, scale separation

Local dynamics from N-S

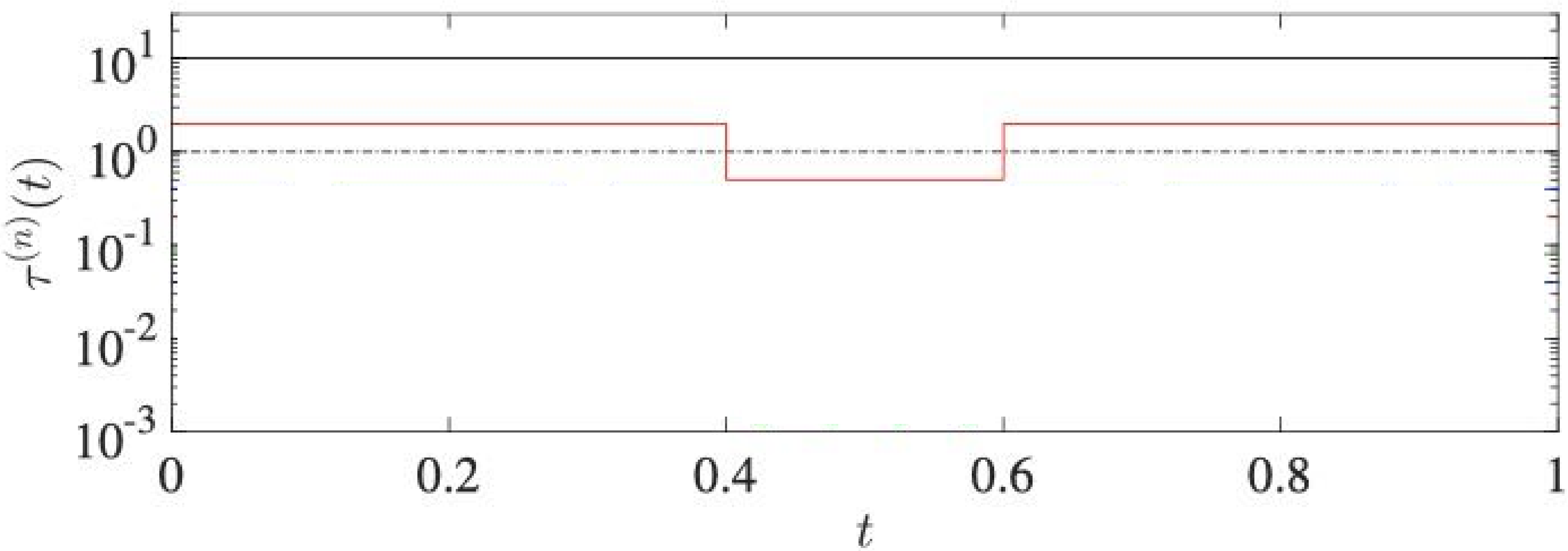
$$\frac{dA}{dt} = -\mathbf{A}^2 + .. \mathbf{F}$$



An illustrative temporal cascade toy problem:

Luo et al. (2022, Phys. Rev. Fluids, in press)

$$\tau^*(t^*) = \begin{cases} 2.0 & 0 \leq t^* < 0.4 \quad \text{and} \quad 0.6 \leq t^* < 1 \\ 0.5 & 0.4 \leq t^* < 0.6. \end{cases}$$



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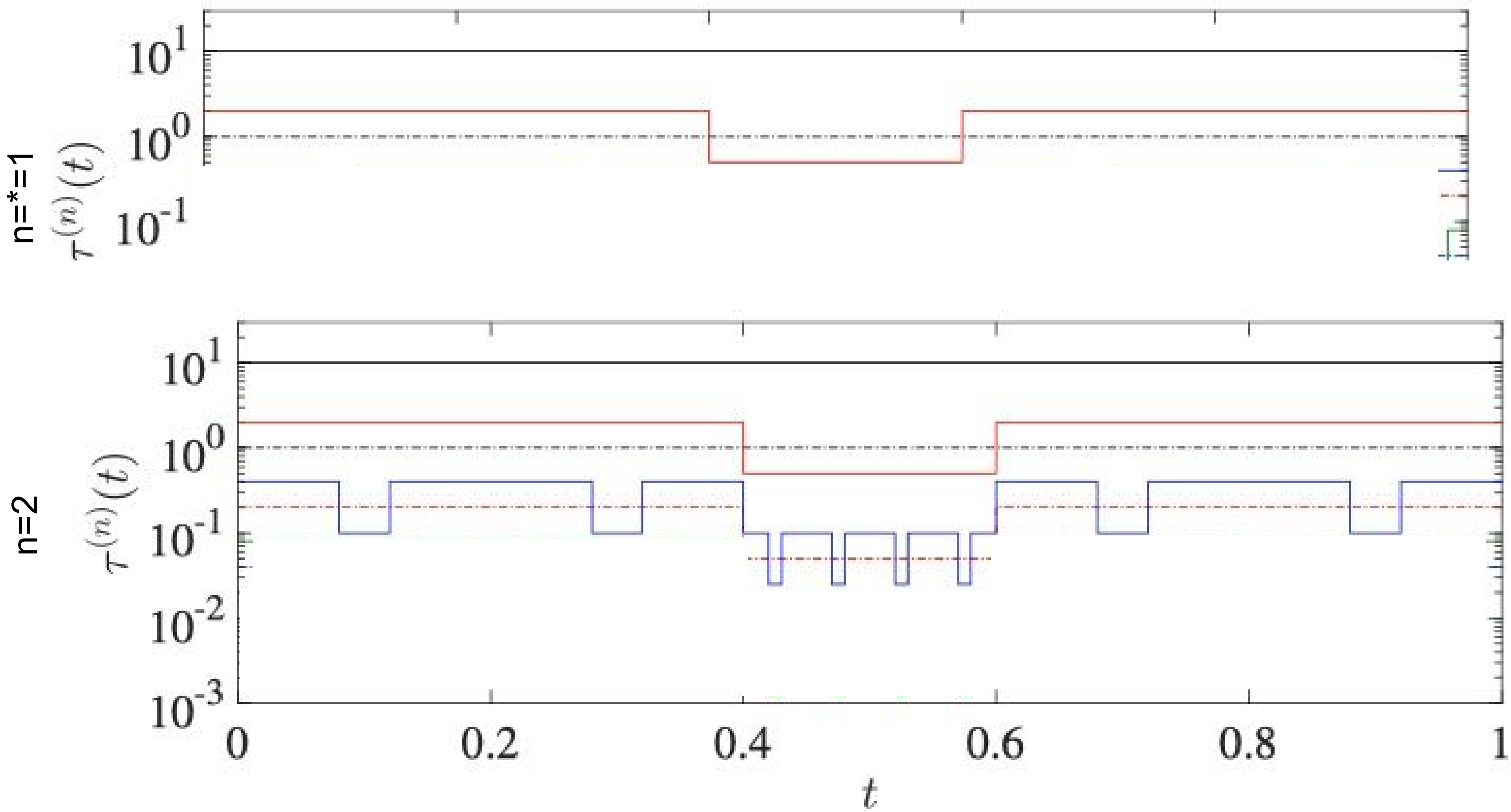

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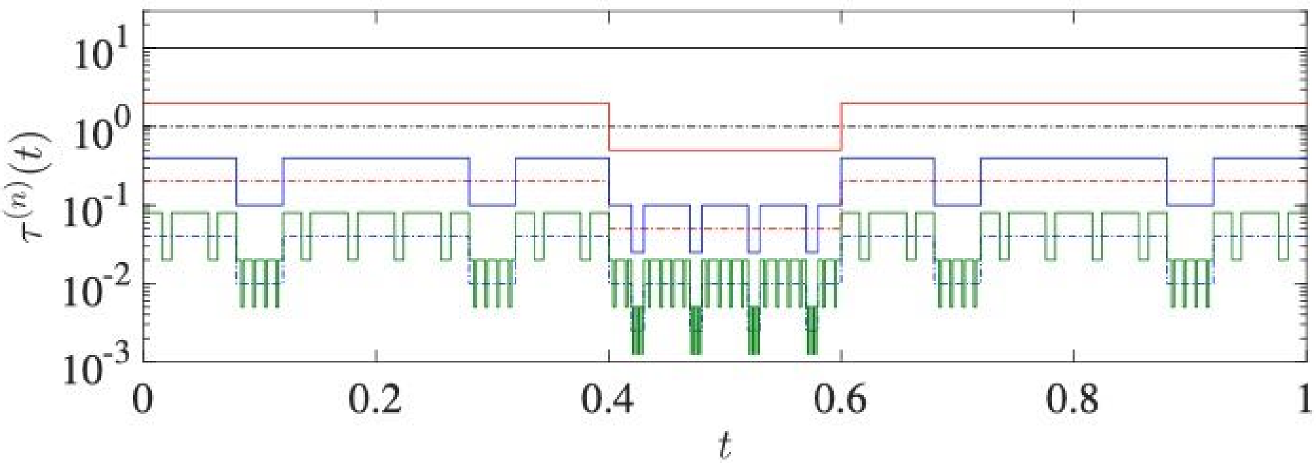

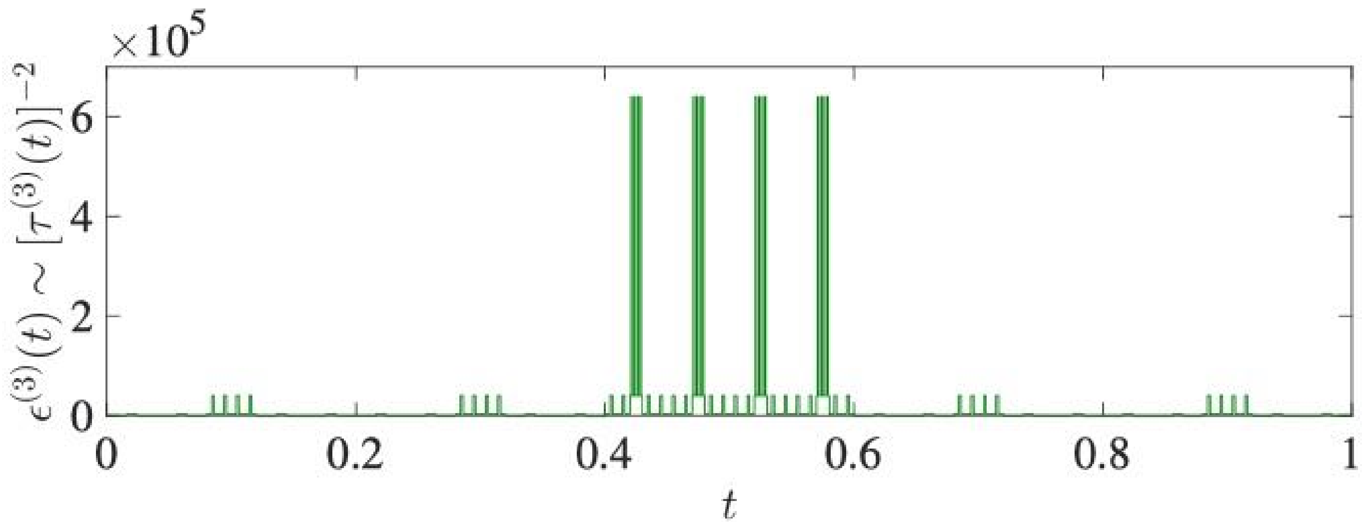

Same process but based on master level RDGF model:

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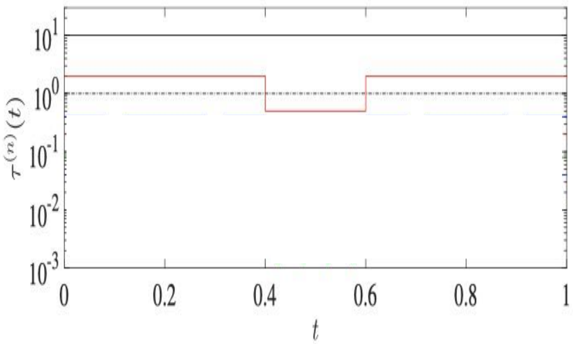
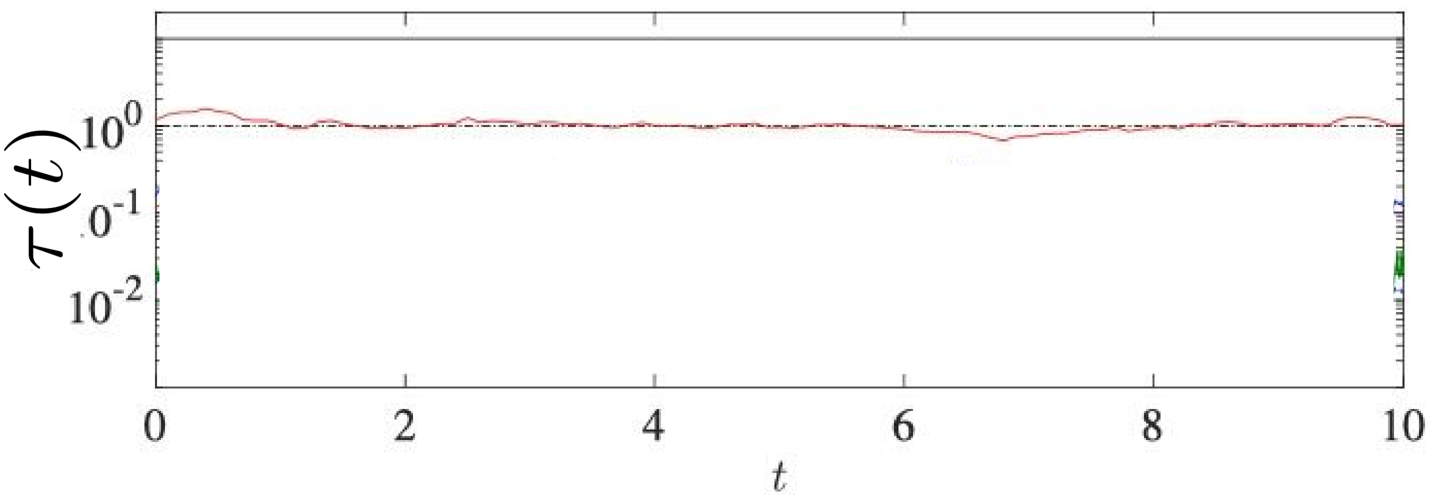


$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3}A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \tau_K) \right] dt + dF_{ij}(\tau_K) \qquad \tau(t) = 1/\sqrt{S_{ij}(t)S_{ij}(t)}$$



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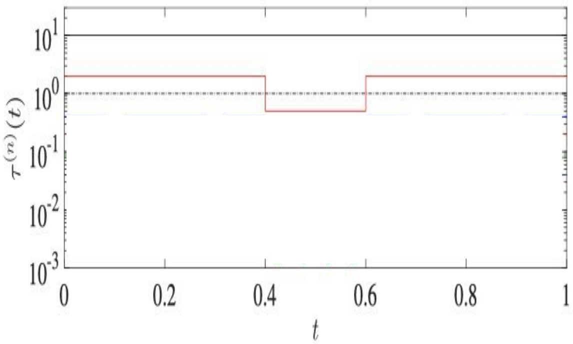
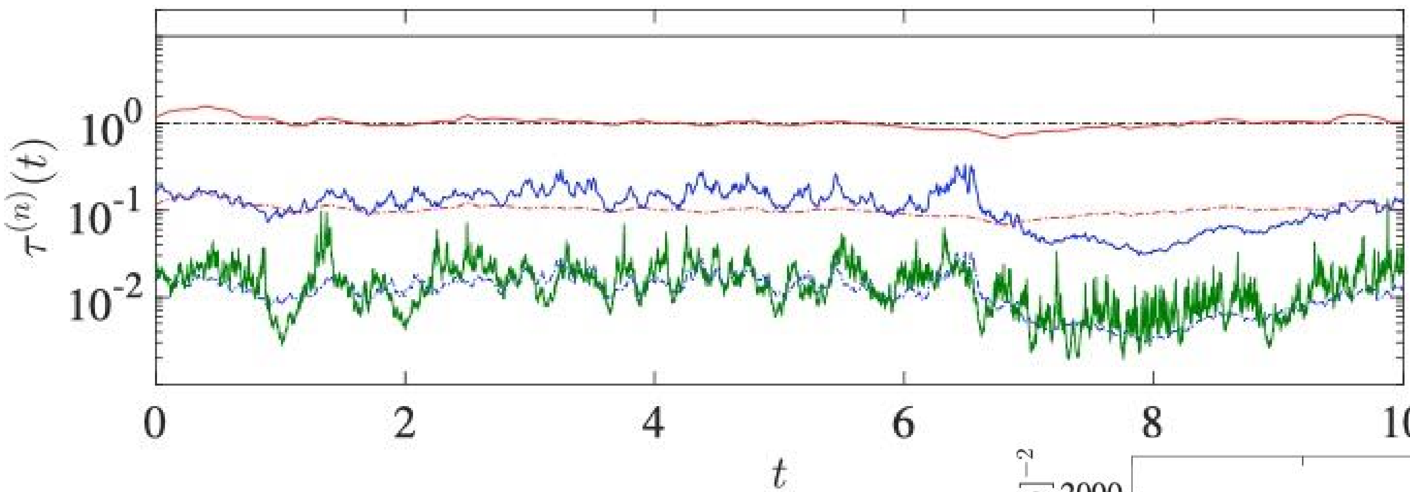
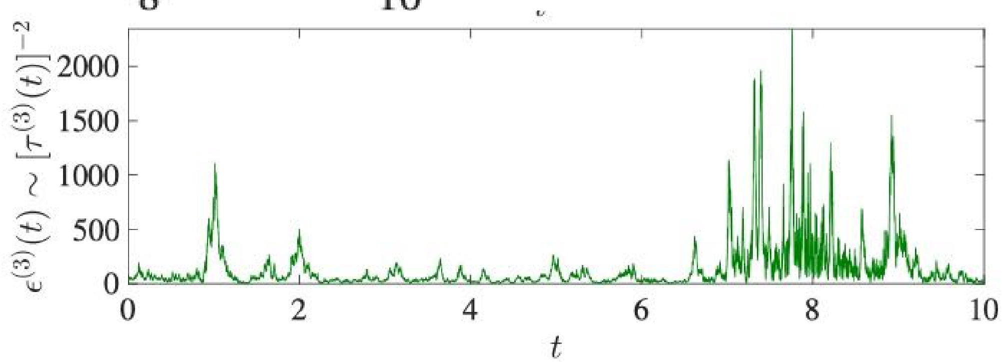


$$dA_{ij} = \left[-\left(A_{ik}A_{kj} - \frac{1}{3}A_{pq}A_{pq}\delta_{ij} \right) + h_{ij}(\mathbf{A}, \tau_K) \right] dt + dF_{ij}(\tau_K) \qquad \tau(t) = 1/\sqrt{S_{ij}(t)S_{ij}(t)}$$



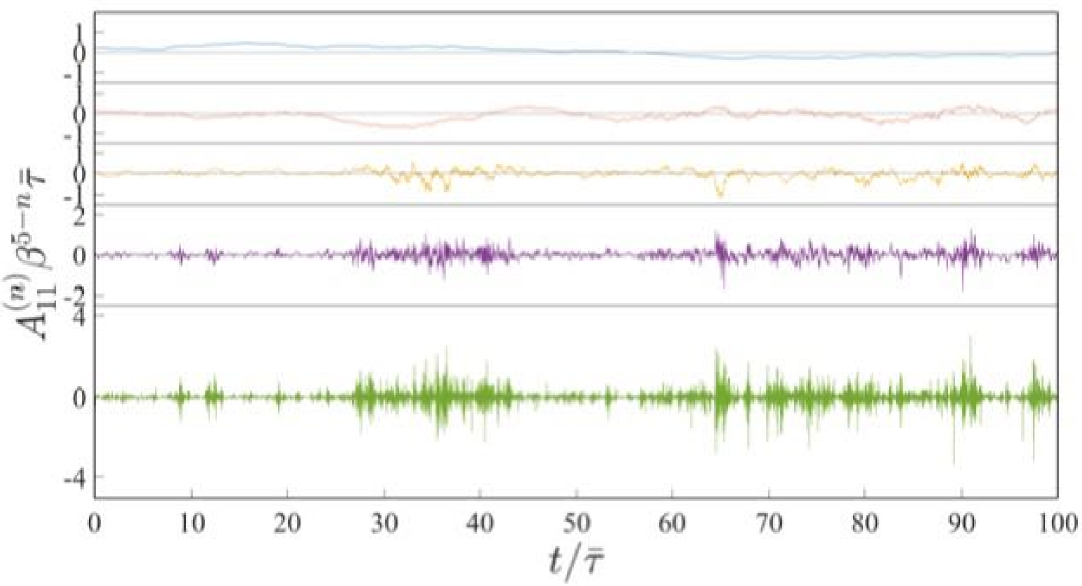


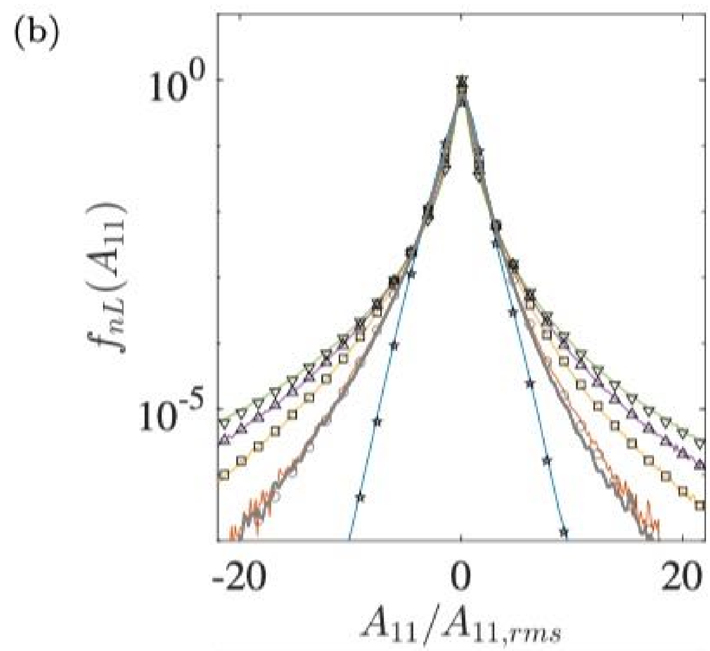
Results: Time scale modulation

$$\beta = 10$$

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \quad \tau_n(t) = \frac{1}{\beta} \left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)} \right)^{-1/2}$$

Longitudinal gradients



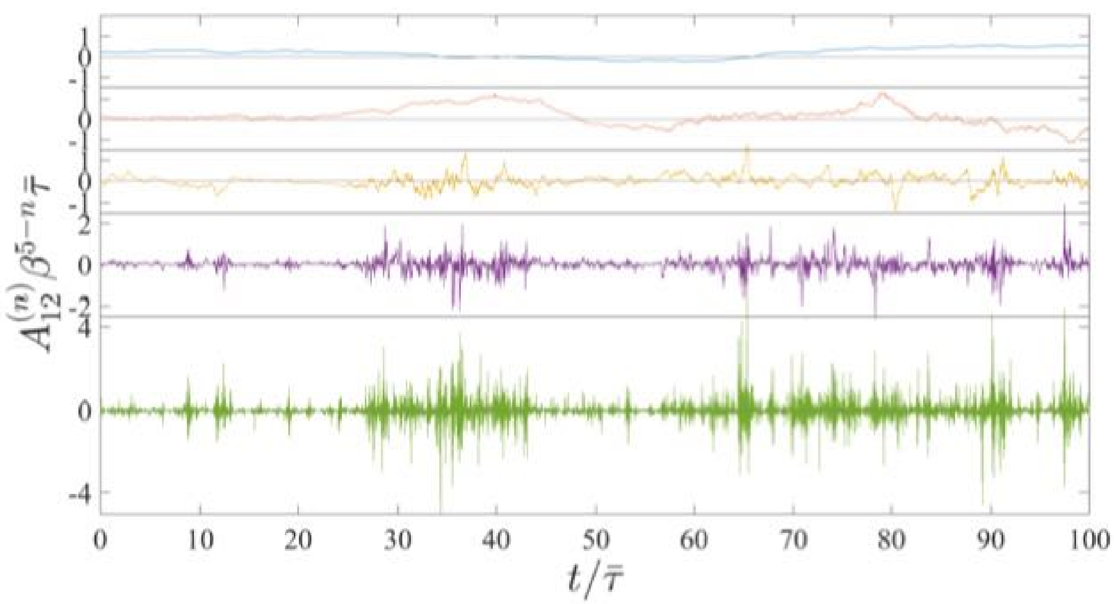


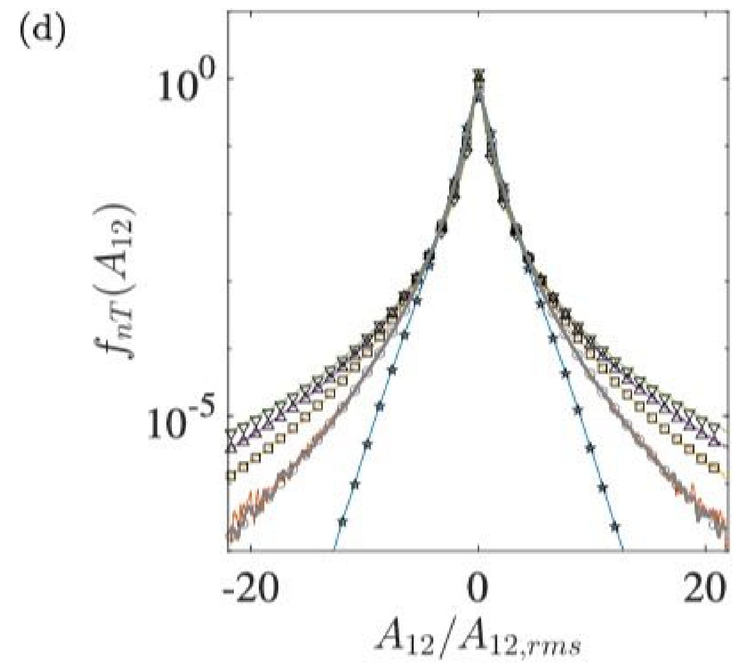
Results: Time scale modulation

$$\beta = 10$$

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \boldsymbol{\tau}_n) - \frac{\dot{\boldsymbol{\tau}}_n}{\boldsymbol{\tau}_n} A_{ij} \right] dt + dF_{ij}(\boldsymbol{\tau}_n) \quad \tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)}\right)^{-1/2}$$

Transverse gradients





Extension to arbitrary high and intermediate Reynolds – convolutions of PDFs (products of characteristic functions..)

Luo et al. (2022, Phys. Rev. Fluids, in press)

$$\epsilon^{(n)}(t) = \tau_n(t)^2 \epsilon^* \left(\int_0^t \frac{1}{\tau_n(t')} dt' \right)$$

$$y_n = \ln(\tau_n), \qquad y^* = \ln(\tau^*)$$

$$f_{n\epsilon}(\mathcal{E}) = \int_0^{+\infty} \mathcal{T}^2 \, f_{*\epsilon}(\mathcal{T}^2 \mathcal{E}) \, f_{n\tau}(\mathcal{T}) d\mathcal{T}$$

$$f_{ny}(\mathcal{Y} + (n-1)\ln\beta) = \mathcal{F}^{-1}\left[\{\mathcal{F}\left[f_{*y}(\mathcal{Y})\right]\}^{n-1} \right]$$

works for fractional n (continuous Re)

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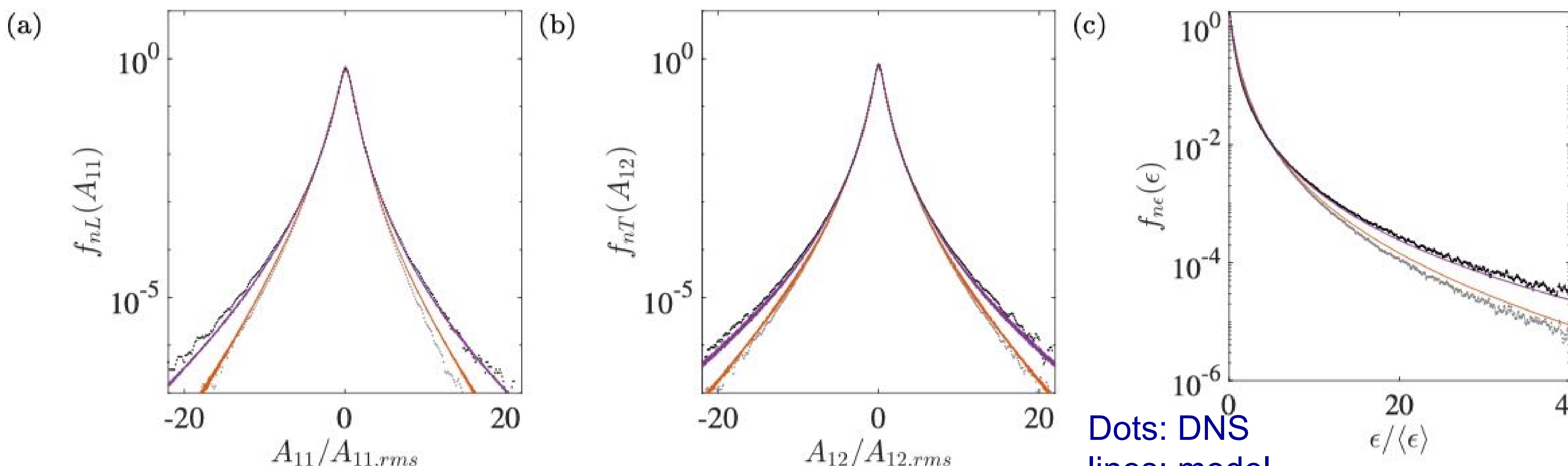


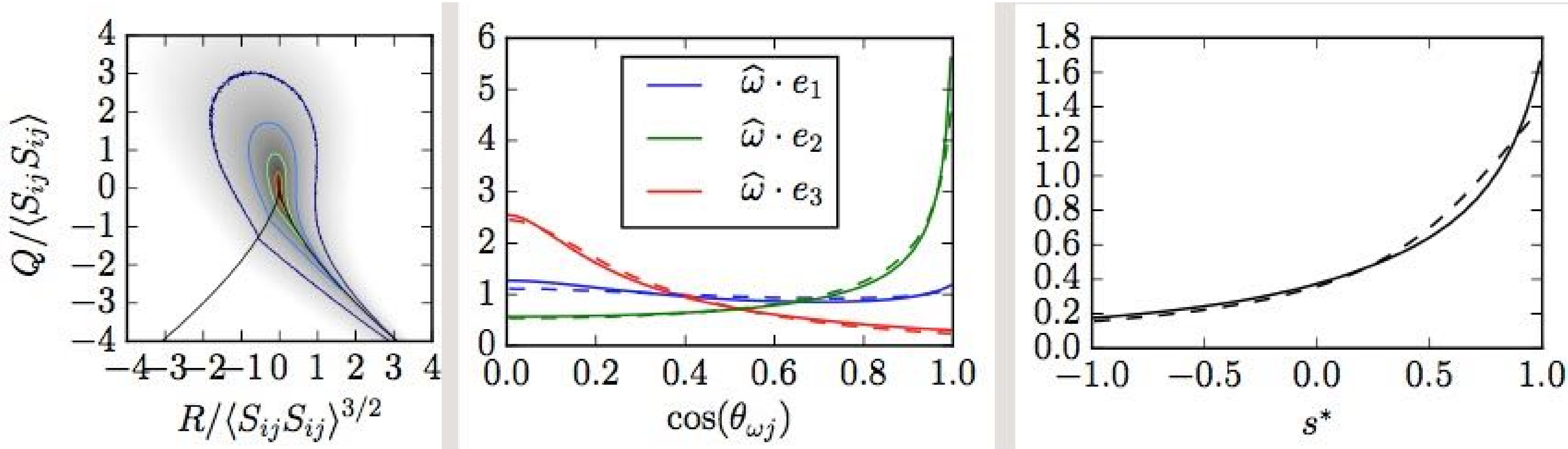
Dots: DNS
lines: model

Probability density functions of A_{11} (a), A_{12} (b) and dissipation (c) of $n = 1.85$ (red solid lines) and $n = 2.33$ (purple solid lines) compared with the $\mathrm{Re}_\lambda = 430$ (grey dot line) and $\mathrm{Re}_\lambda = 1300$ (black dot line) DNS results.

Results: local flow topology

$$\beta = 10$$

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \qquad \tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)} \right)^{-1/2}$$



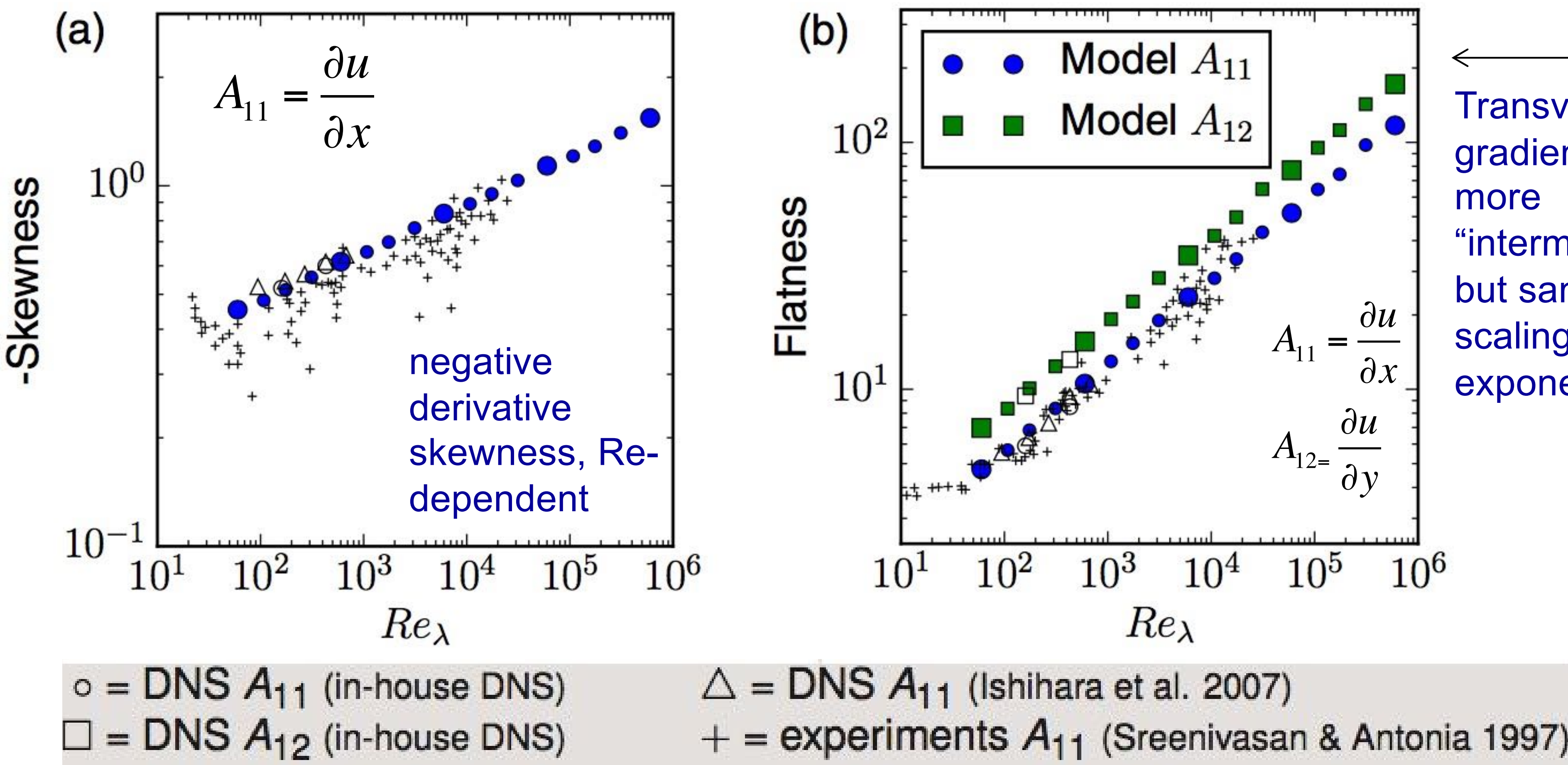
Right two plots: solid lines are model results at $Re_\lambda = 430$, dashed lines are DNS results from Johns Hopkins Turbulence Database: `http://turbulence.pha.jhu.edu/`.

Pick N to correspond to Re_λ=430 (N_{eff}=1.85)

Reynolds number scaling of –S and F

$$\beta = 10$$

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \quad \tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)}\right)^{-1/2}$$



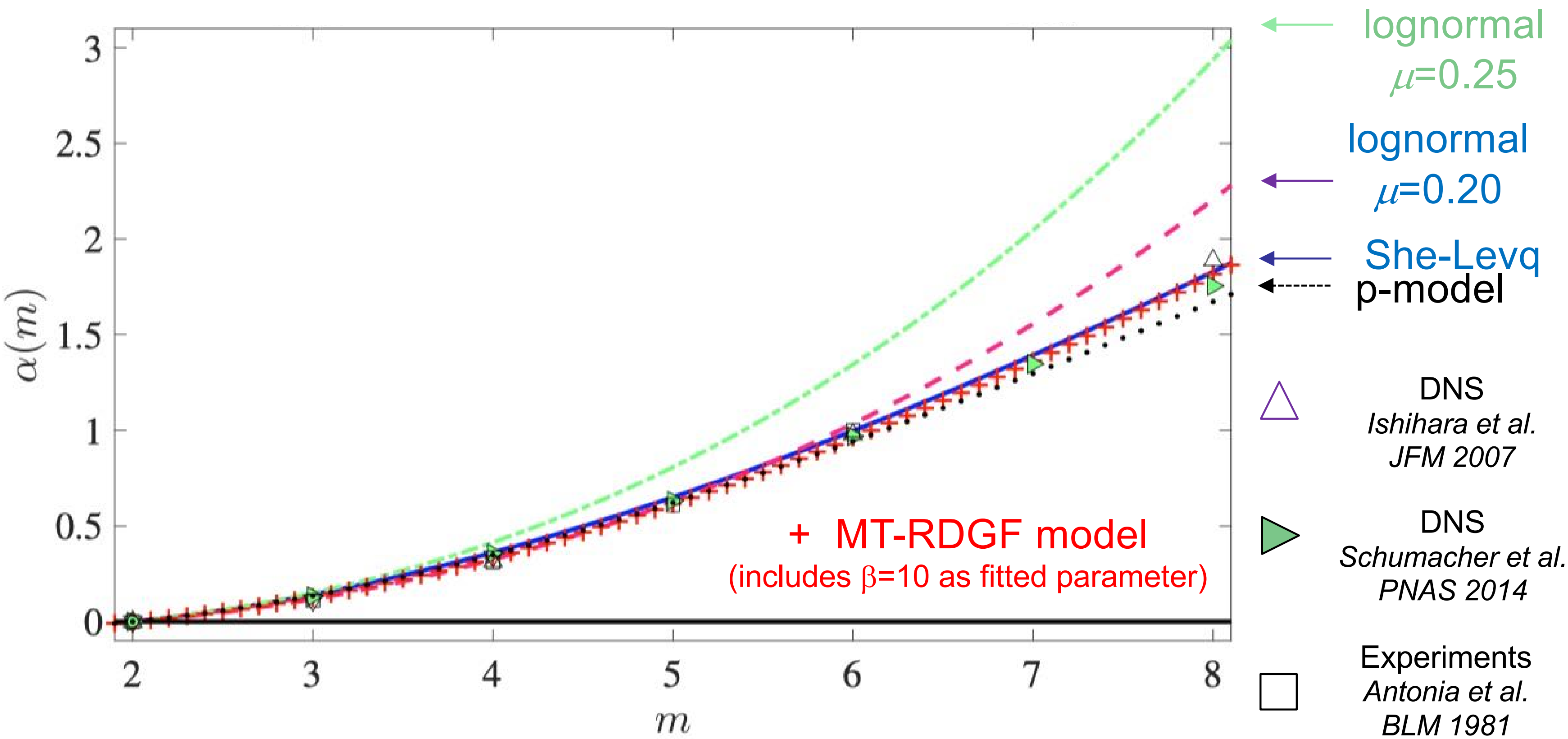
← Transverse gradient more “intermittent” but same scaling exponent

Results: anomalous gradient exponents

Luo et al. (2022, Phys. Rev. Fluids, in press)

$$dA_{ij}^{(n)} = \left[-\left(A_{ik}^{(n)} A_{kj}^{(n)} - \frac{1}{3} A_{pq}^{(n)} A_{pq}^{(n)} \delta_{ij} \right) + h_{ij}^{n}(\mathbf{A}, \tau_n) - \frac{\dot{\tau}_n}{\tau_n} A_{ij} \right] dt + dF_{ij}(\tau_n) \quad \tau_n(t) = \frac{1}{\beta}\left(2 S_{ij}^{(n-1)} S_{ij}^{(n-1)}\right)^{-1/2}$$

$$\frac{\left\langle |A_{11}|^m \right\rangle}{\left\langle |A_{11}|^2 \right\rangle^{m/2}} \sim \mathrm{Re}^{\alpha(m)}$$



Conclusions:

$A_{ij}^{(n)}(t)$

$A_{ij}^{(2)}(t)$

$A_{ij}^{(1)}(t)$

- Low-order model, only 9N SDE
- Can obtain all statistics from **single** “master level”
- Term driving intermittency growth comes from Navier-Stokes (not modeled)
- Physically motivated closure for damping terms (pressure Hessian & viscous):

  Gaussian i.c. + fluid deformation from **A**(t)
- Predicts multiple “geometric” vorticity behaviors
- Predicts intermittency growth with Re
- Requires 4 scalar parameters, τ, D_a, D_s, β:
  - ✓ 3 fixed from self-consistency conditions.

  but 1 still needs matching with data to fit Re dep.

$$\frac{dA_{11}^{(1)}}{dt} = \ldots$$
$$\frac{dA_{12}^{(1)}}{dt} = \ldots$$
$$\frac{dA_{13}^{(1)}}{dt} = \ldots$$

…

…

$$\frac{dA_{33}^{(N)}}{dt} = \ldots$$

$9 \times N$ dof

$\mathrm{Re}_\lambda = 60{,}000,$

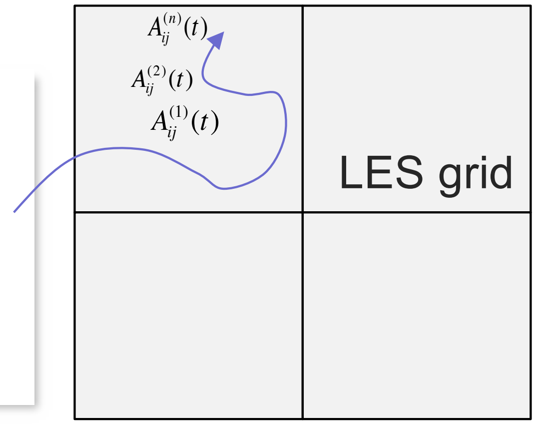
$N = 4$

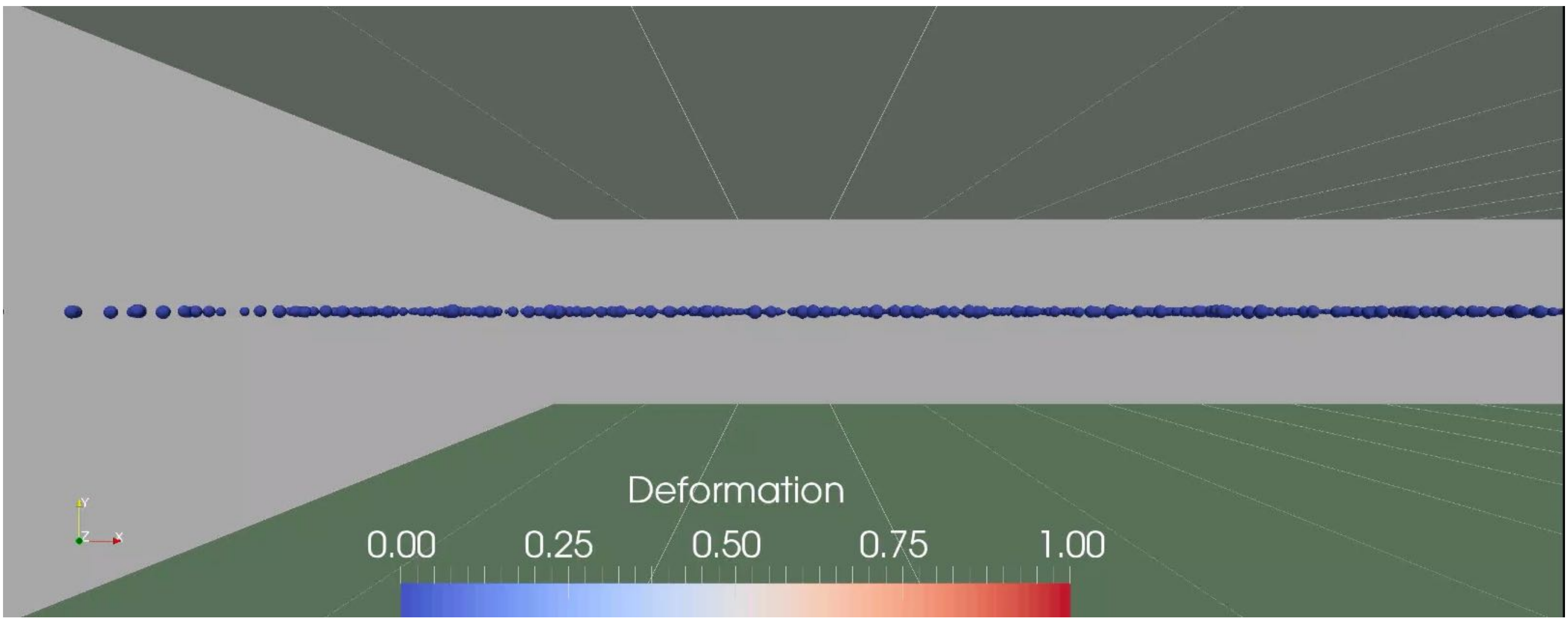
$\rightarrow$ 36 dof

Statistics:
1 level enough
8 dof

Engineering applications

- LES of sub Kolmogorov scale droplets' deformation in turbulent channel flow
- See Johnson & CM (JFM 2017, single level)





Thank you – questions?

Thanks again to:

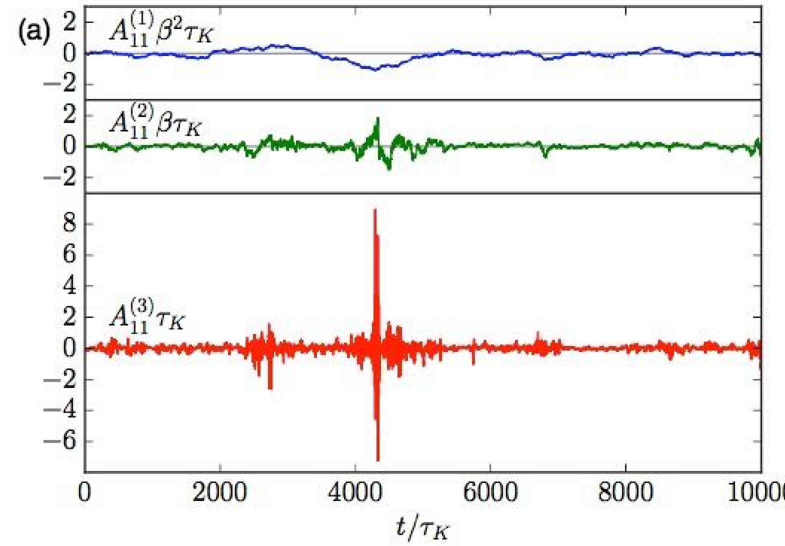
Yi Li

Laurent Chevillard

Michael Wilczek

Perry Johnson

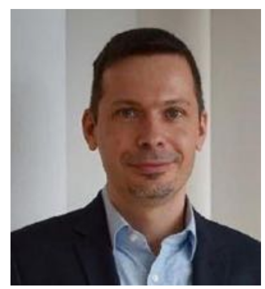
Luo Hao



Yi

Laurent

Michael



Perry

National Science Foundation
WHERE DISCOVERIES BEGIN

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