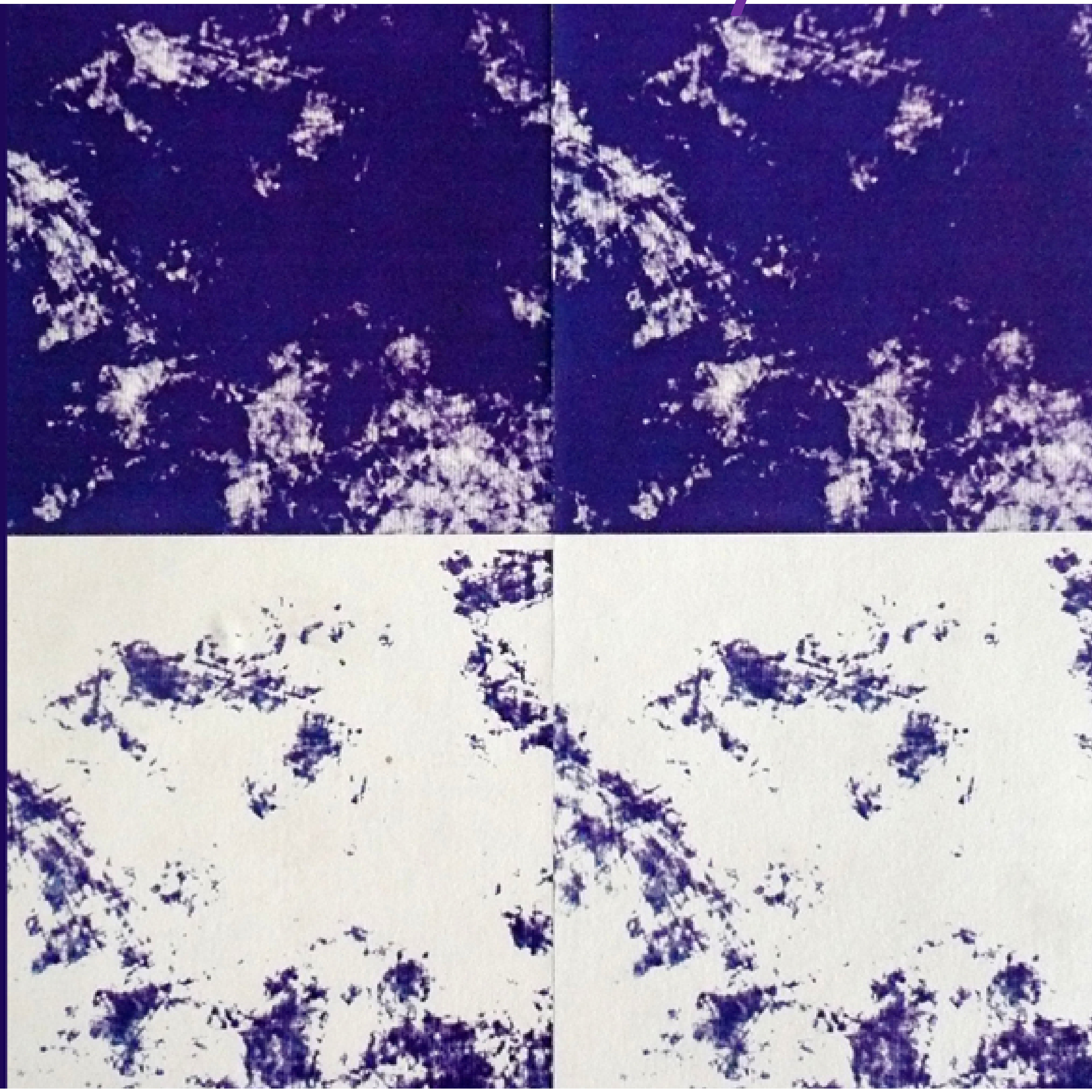
Multifractals, Intermittency, Spectra and Climate Variability Across Scales

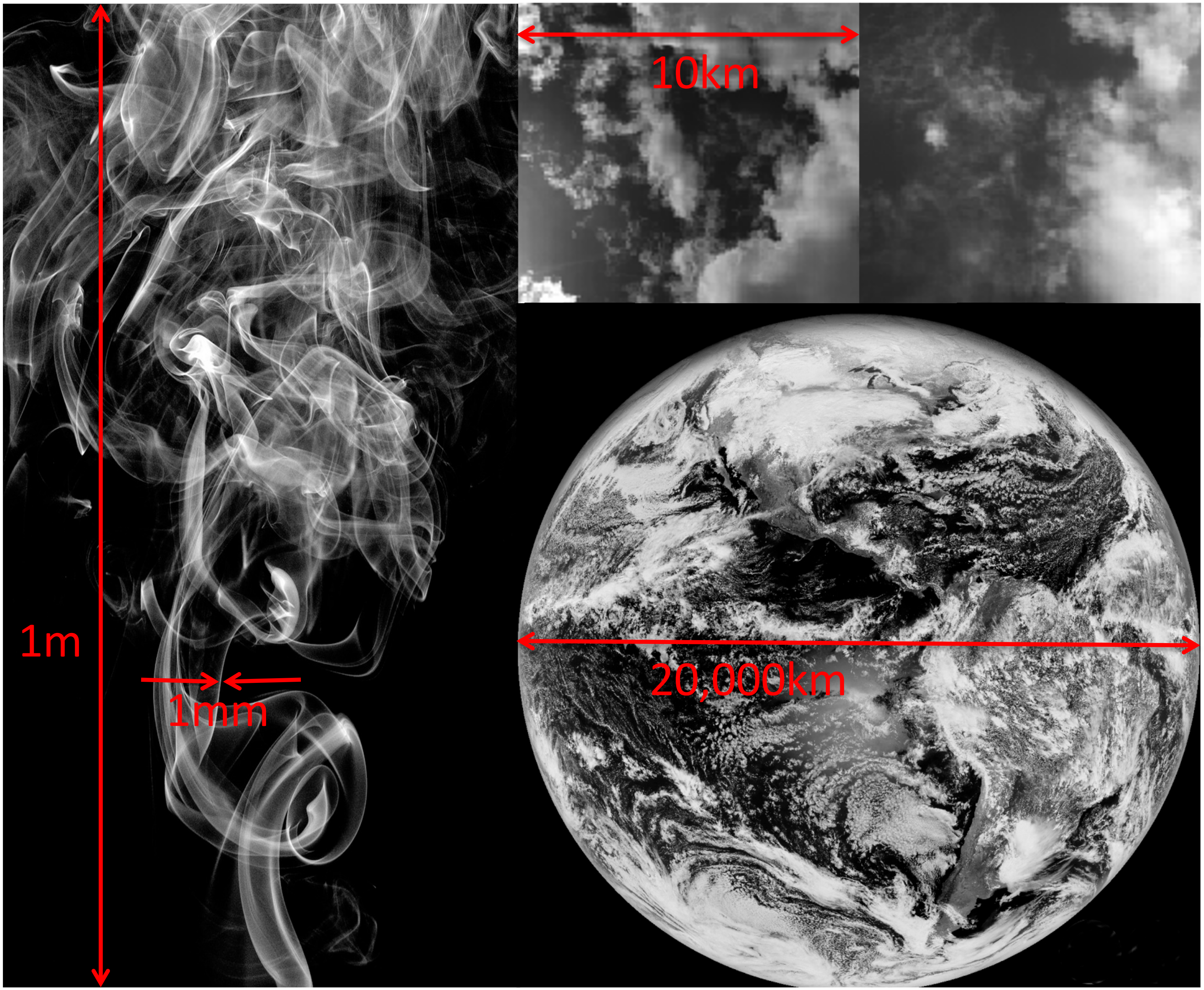


Shaun Lovejoy,
Physics, McGill U., Montreal, Canada

EGU NP Campfire, January 18, 2022

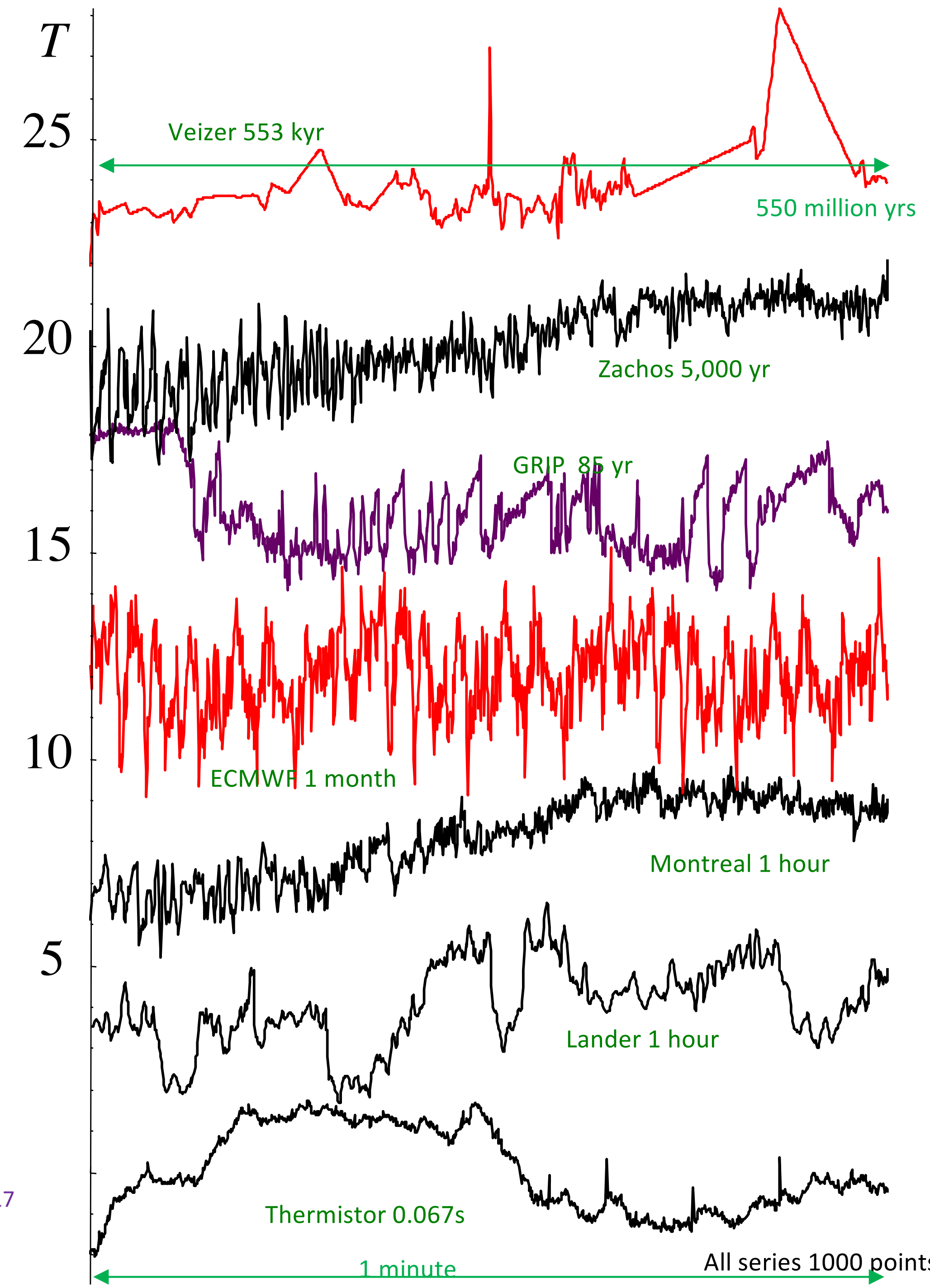
CLIMATE VARIABILITY ACROSS SCALES:
FROM CENTURIES TO MILLENNIA

A range of a billion: 0.1mm – 10,000km



Range in time scales: a billion billion: 0.001s – 4.5 billion years

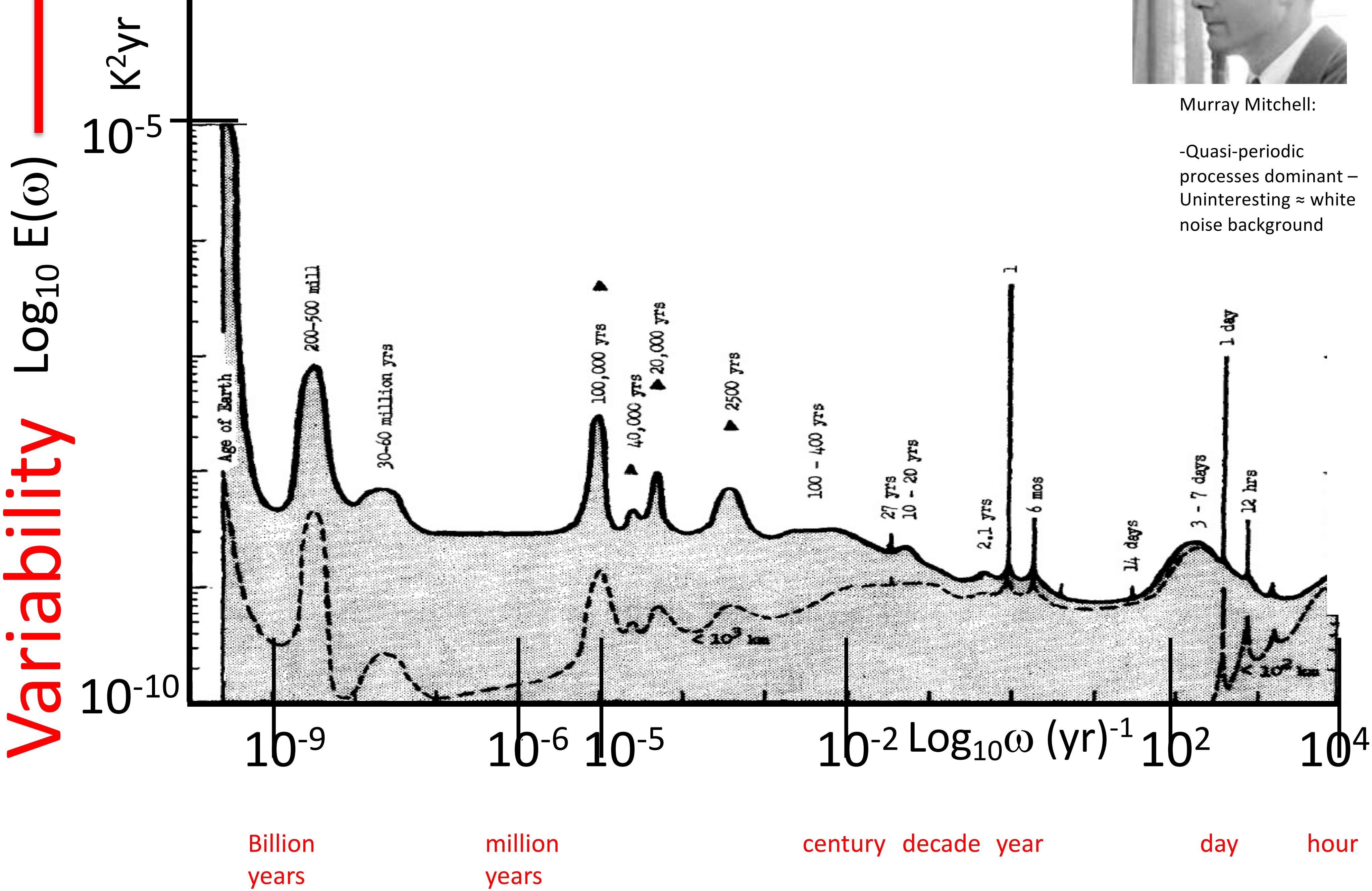
In this plot: range of scales >10^{17}



How to understand variability over range of billions (space), billion billions (time) ?

Answer 1:
(Still) dominant Scalebound view:
Ideological and Qualitative

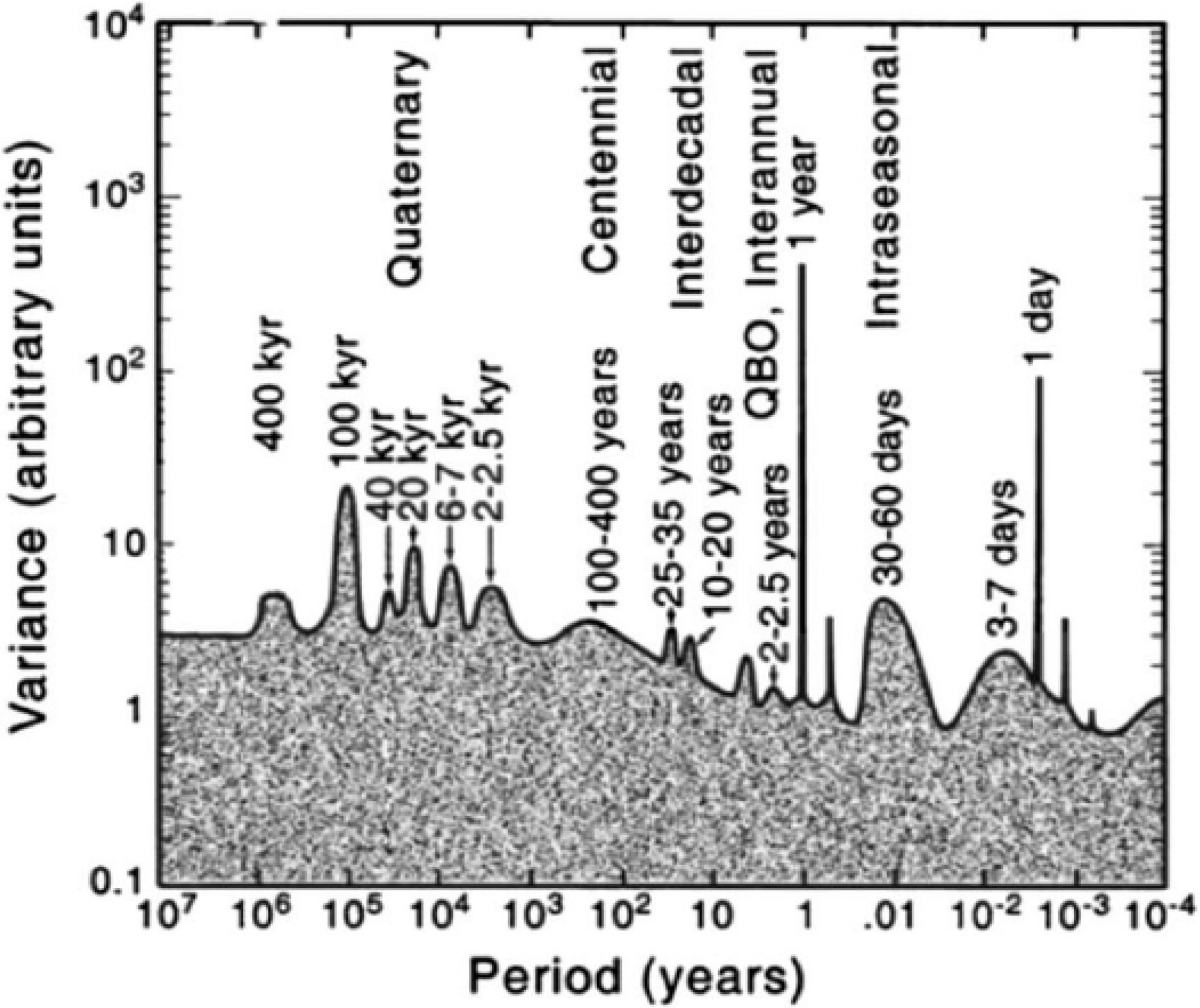
"Educated Guess" Mitchell's (1976)



Murray Mitchell:

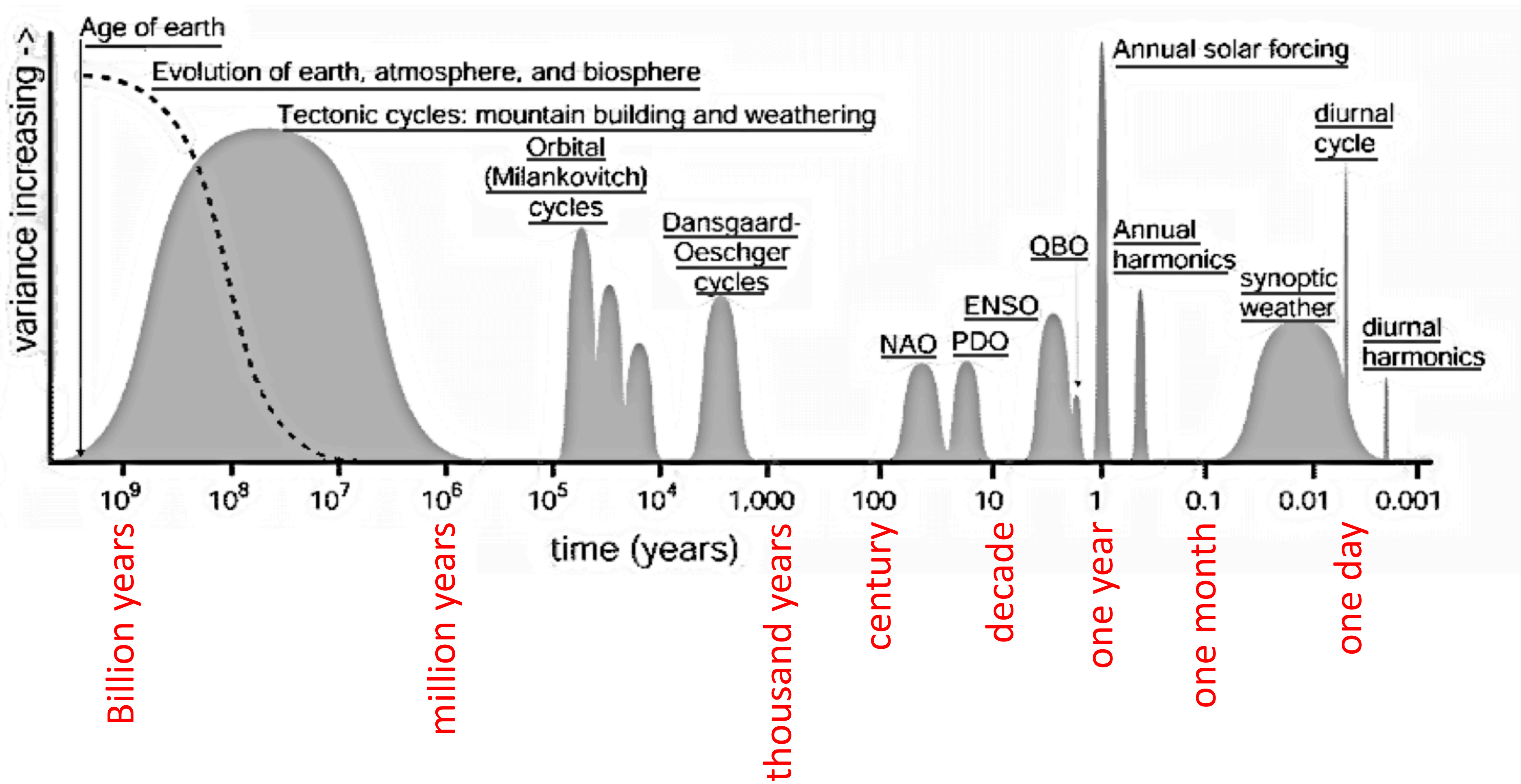
-Quasi-periodic processes dominant –
Uninteresting ≈ white noise background

“Artist’s Rendering” (2002-2020)



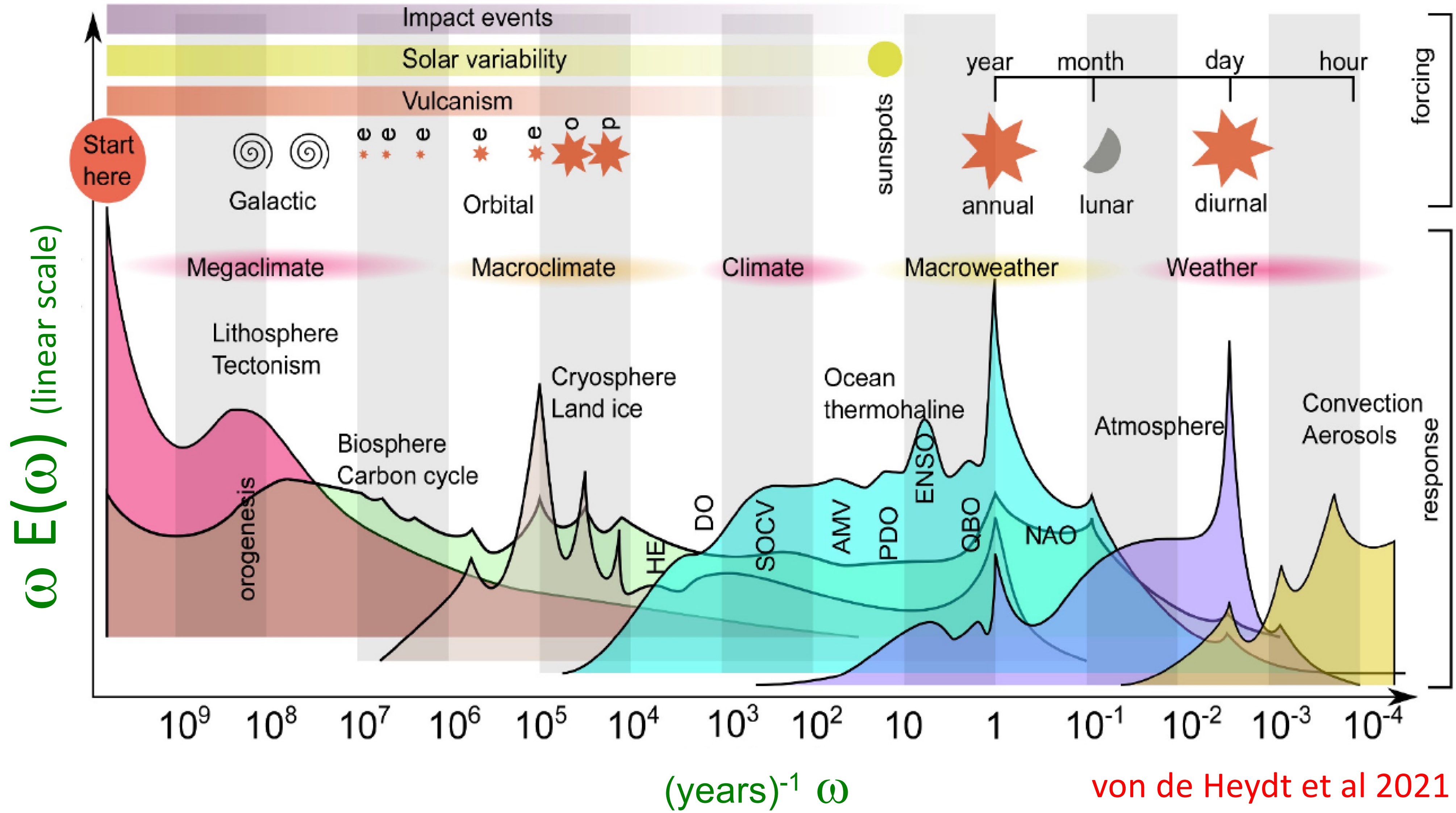
Ghil 2002,
Reprinted in
Ghil, Lucarini 2020

"Mental Model" 2015



"Inspired by Mitchell":
National Oceanographic and Atmospheric Administration (NOAA) website 2015

"Conceptual Landscape" 2021



Relative spectrum after "removal of background" by dividing by $1/\omega$ spectrum

The scalebound view over time

↓

Educated Guess (1976)

↓

Artist's Rendition (2002, 2020)

↓

Mental Model (2015)

↓

Conceptual Landscape (2021)

Background : not relevant/important, ignored

How to understand variability over range of billions (space), billion billion (time) ?

Answer 2:

Scaling view:

Taking the data (and background) seriously

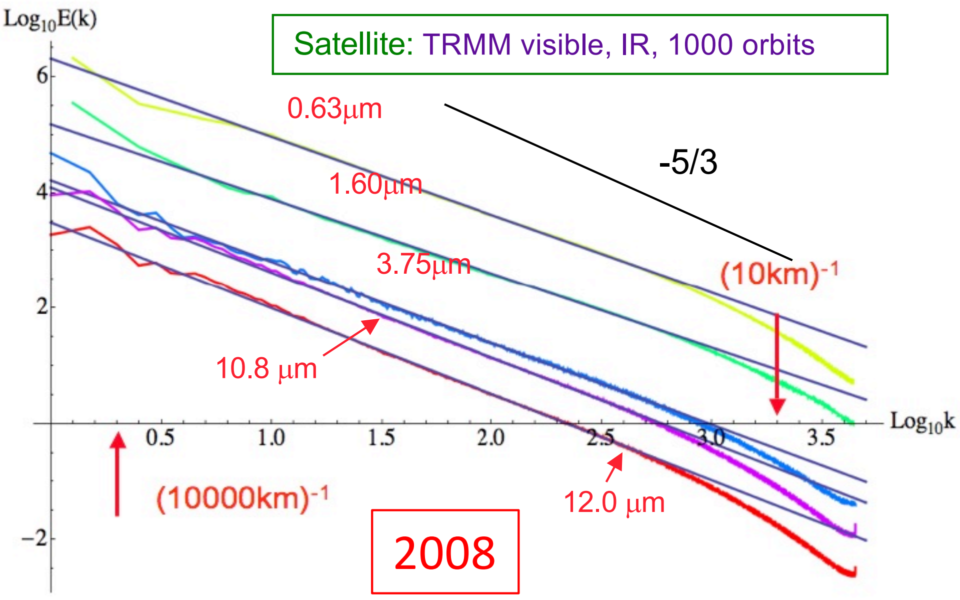
Quantitative

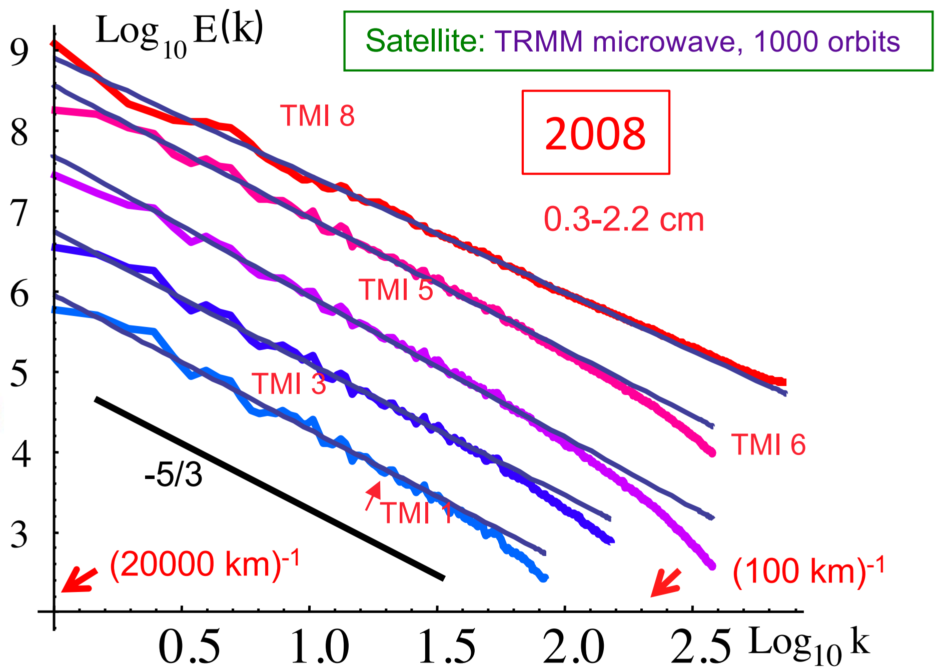
The impact of data

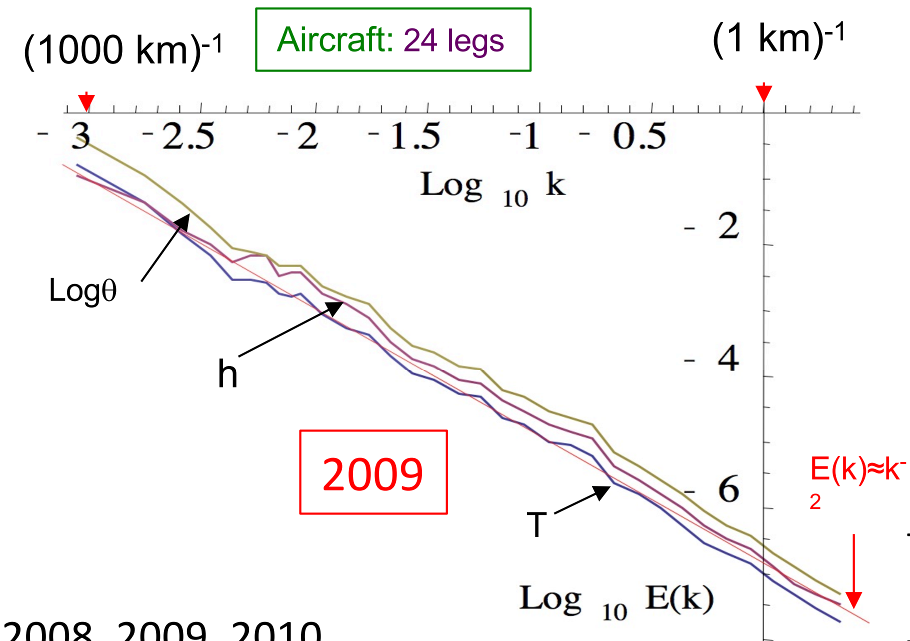
Scaling of spectra

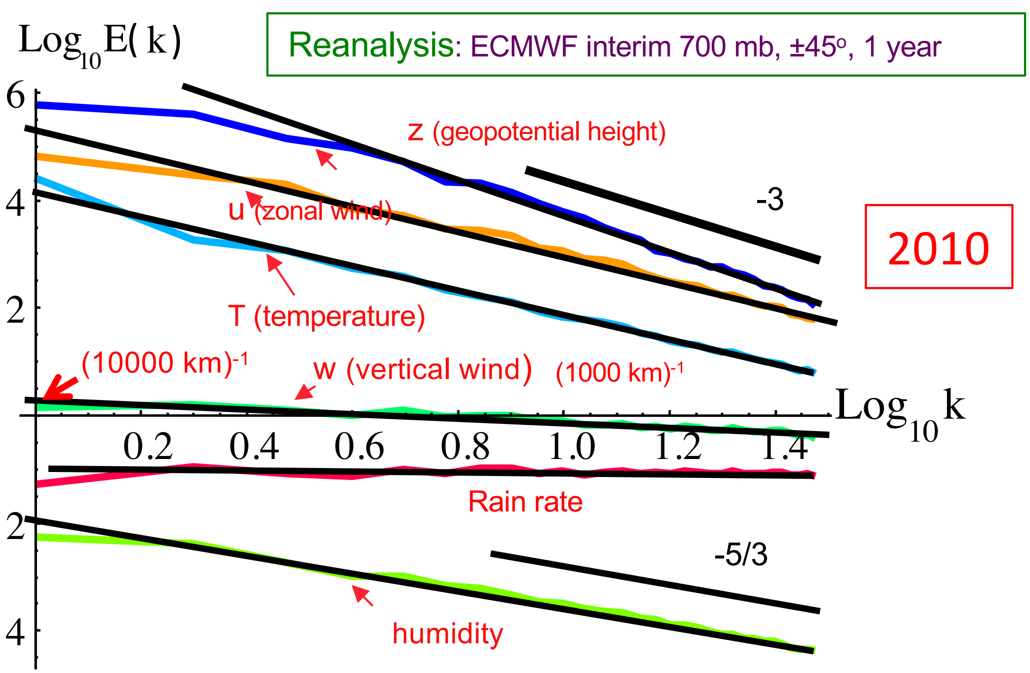
2000-2010: Planetary scale Horizontal Scaling

$$E(k) = k^{-\beta}$$







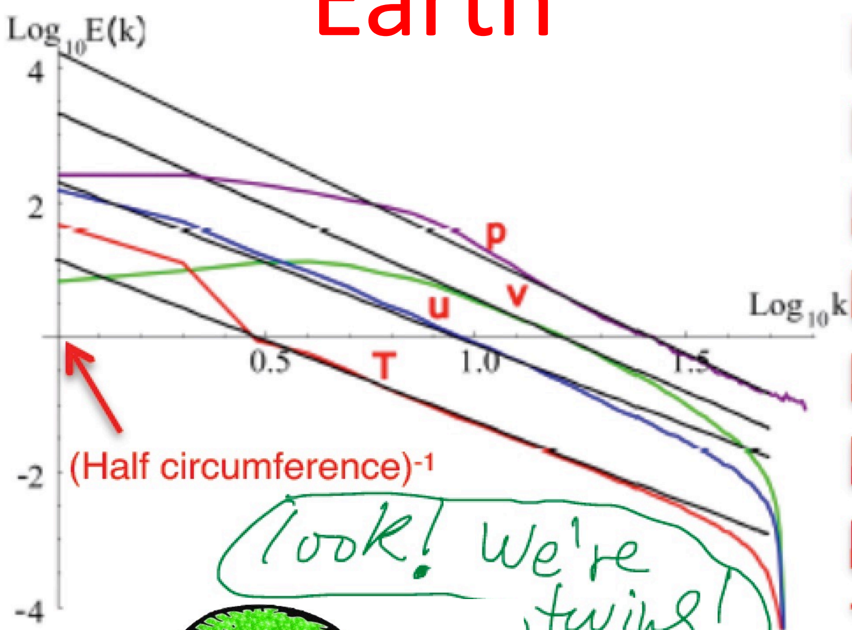


L et 2008, 2009, 2010

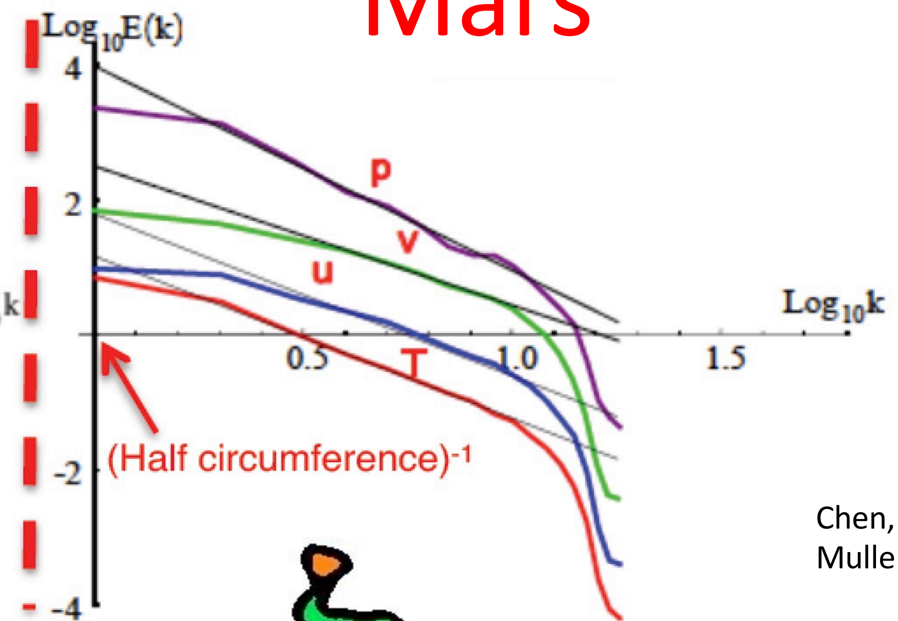
2016: Earth Versus Mars

Comparing spectra from reanalyses

Earth



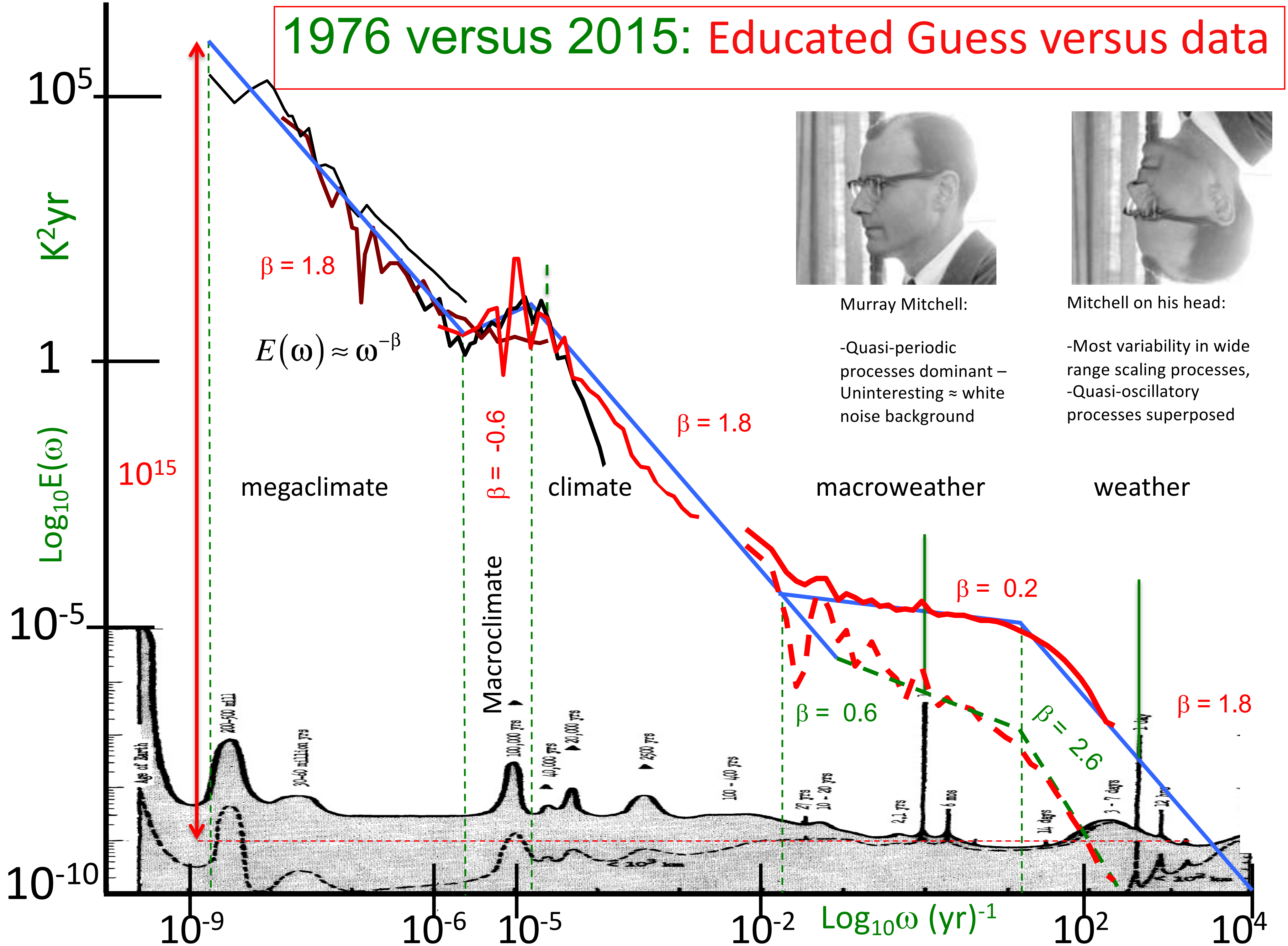
Mars



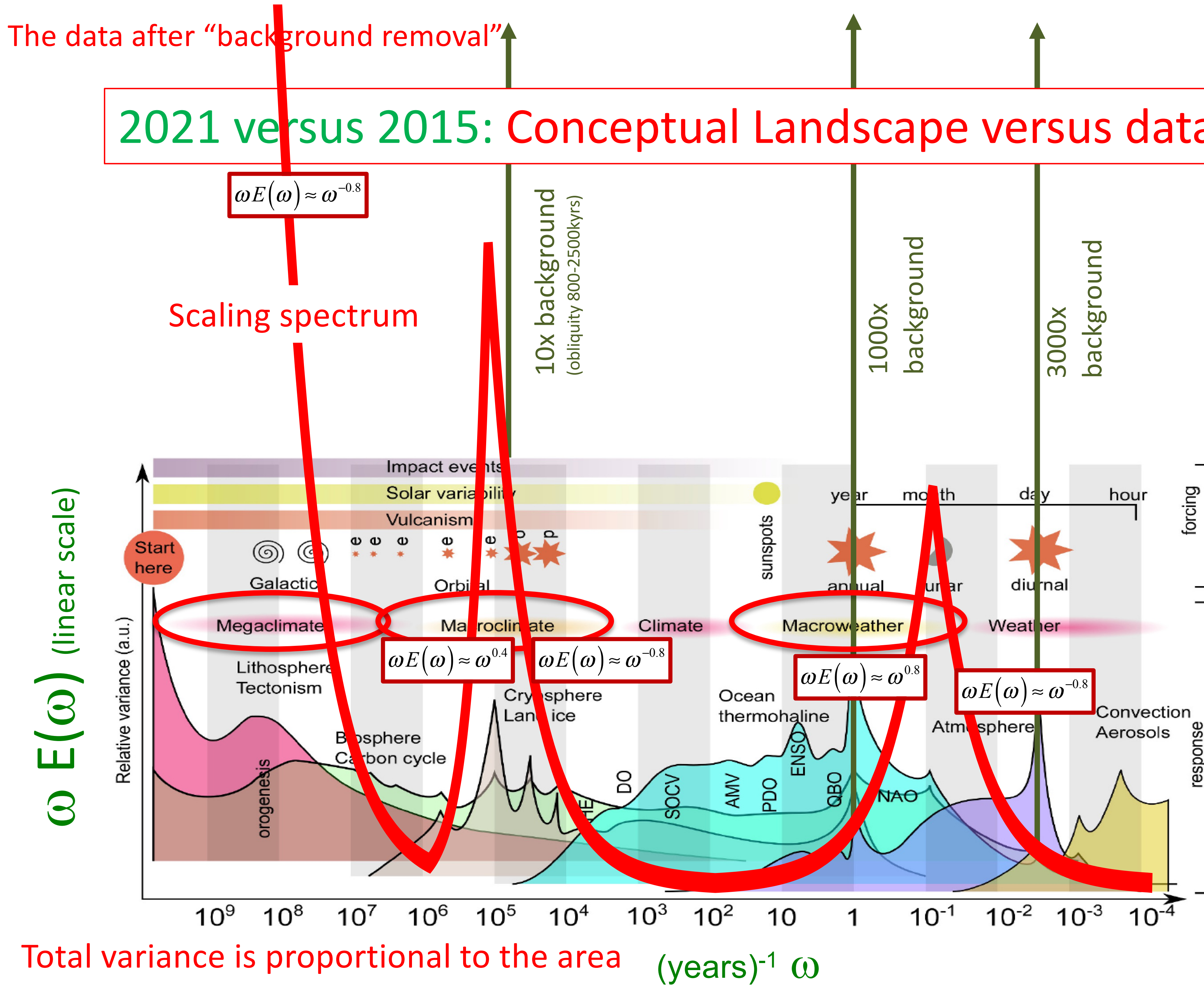
Chen, Lovejoy, Muller 2016

"solar system" universality

1976 versus 2015: Educated Guess versus data



L 2015: A voyage through scales, a missing quadrillion and why the climate is *not* what you expect, **Climate Dynamics**



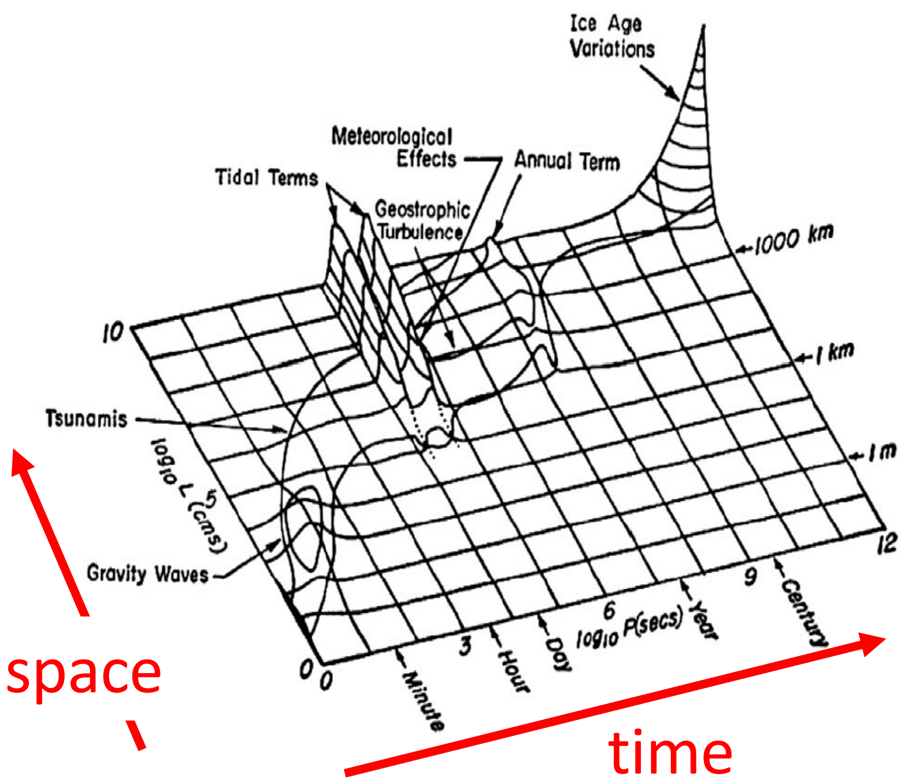
The data after "background removal"
2021 versus 2015: Conceptual Landscape versus data
$\omega E(\omega) \approx \omega^{-0.8}$
Scaling spectrum
10x background
(obliquity 800-2500kyrs)
1000x background
3000x background
Impact events
Solar variability
Vulcanism
sunspots
year
month
day
hour
forcing
Start here
Galactic
Orbital
annual
diurnal
Megaclimate
Macroclimate
Climate
Macroweather
Weather
$\omega E(\omega) \approx \omega^{0.4}$
$\omega E(\omega) \approx \omega^{-0.8}$
$\omega E(\omega) \approx \omega^{0.8}$
$\omega E(\omega) \approx \omega^{-0.8}$
Lithosphere
Tectonism
Biosphere
Carbon cycle
Cryosphere
Land ice
Ocean
thermohaline
Atmosphere
Convection
Aerosols
orogenesis
DO
SOCV
AMV
PDO
ENSO
QBO
NAO
response
Relative variance (a.u.)
ω E(ω) (linear scale)
10^9 10^8 10^7 10^6 10^5 10^4 10^3 10^2 10 1 10^{-1} 10^{-2} 10^{-3} 10^{-4}
Total variance is proportional to the area
(years)$^{-1}$ ω

Space-time “Stommel” diagrams

Scalebound view

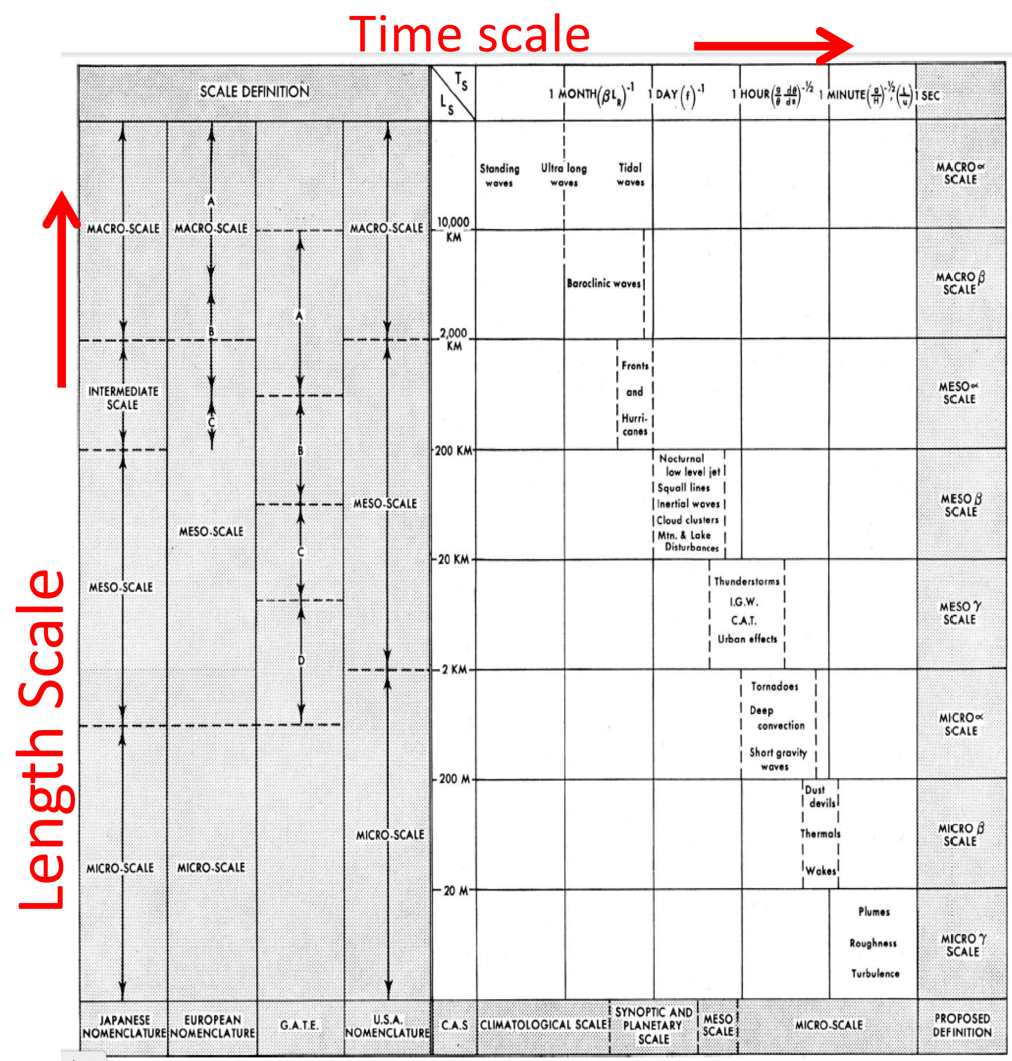
Ocean 1963

Stommel 1963



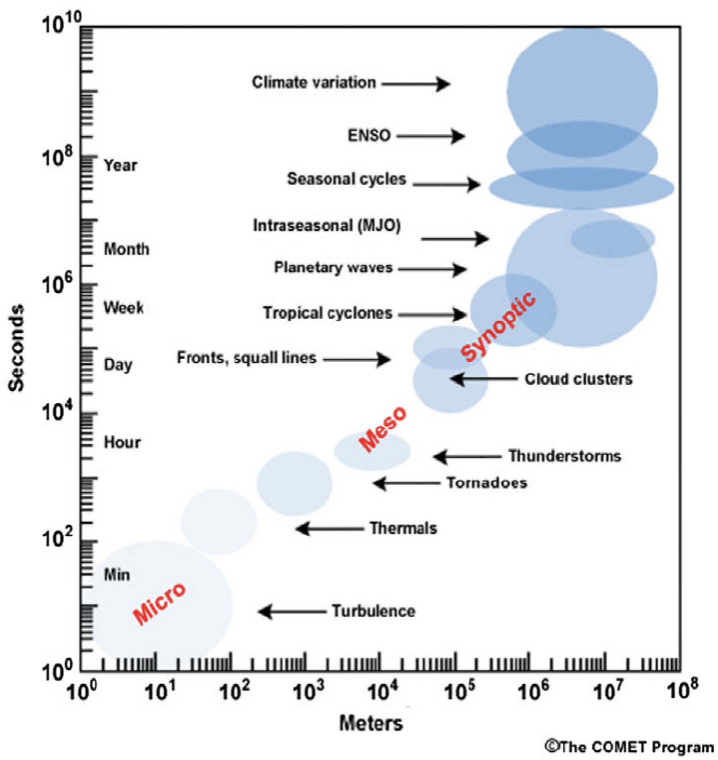
Atmosphere 1976

Orlanski 1976



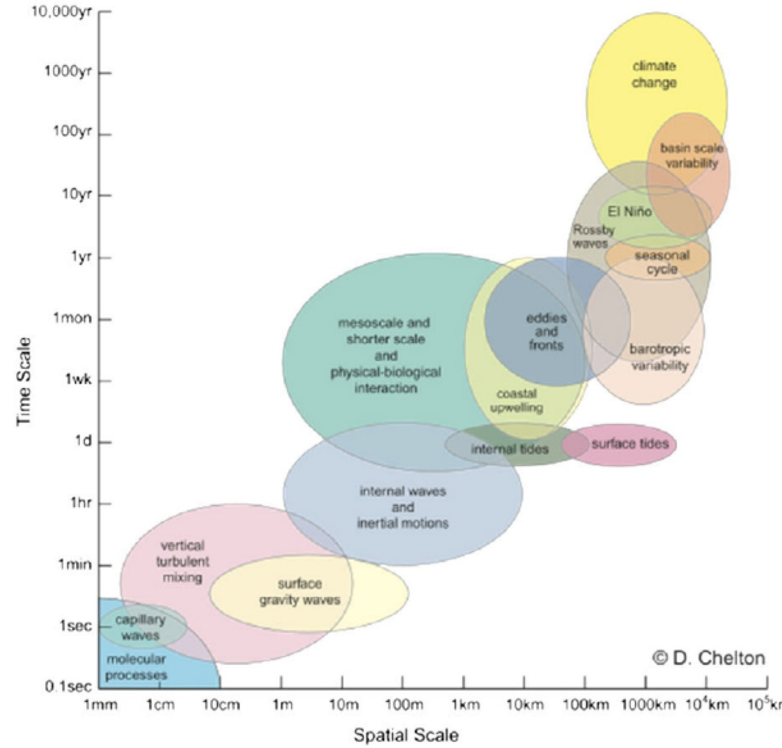
Space-time Diagrams in 2020

Atmosphere



(c)

Ocean



From Ghil, Lucarini 2020 (courtesy D. Chelton)

Space-time Diagrams: The impact of scaling

lifetime

$$\tau = l^{2/3}\varepsilon^{-1/3}$$

Anisotropic extension of Kolmogorov law

$$\Delta v \approx \varepsilon^{1/3} l^{1/3}$$

Horizontal extent

Energy rate density

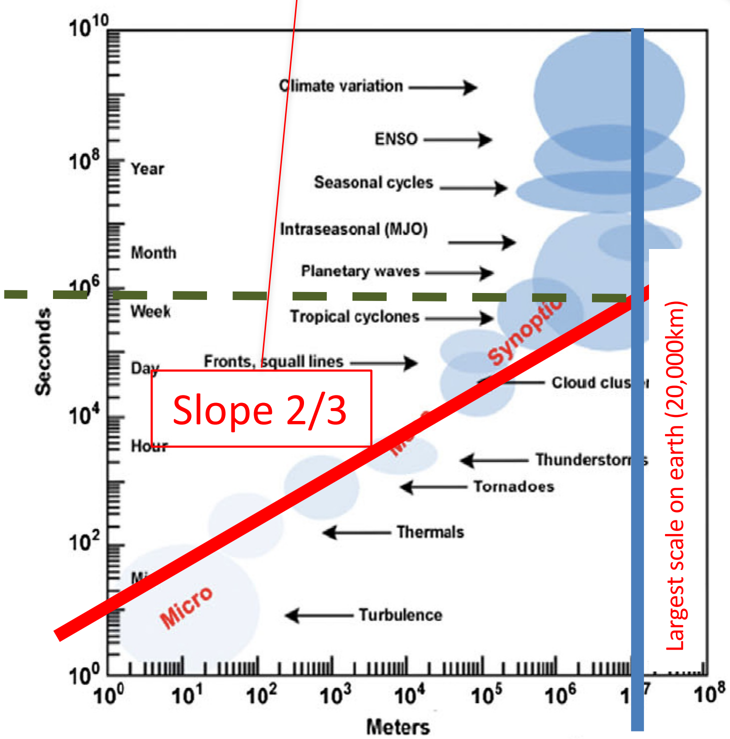
Horizontal only

Velocity Fluctuation

Atmosphere

$\varepsilon \approx 10^{-3}$W/kg

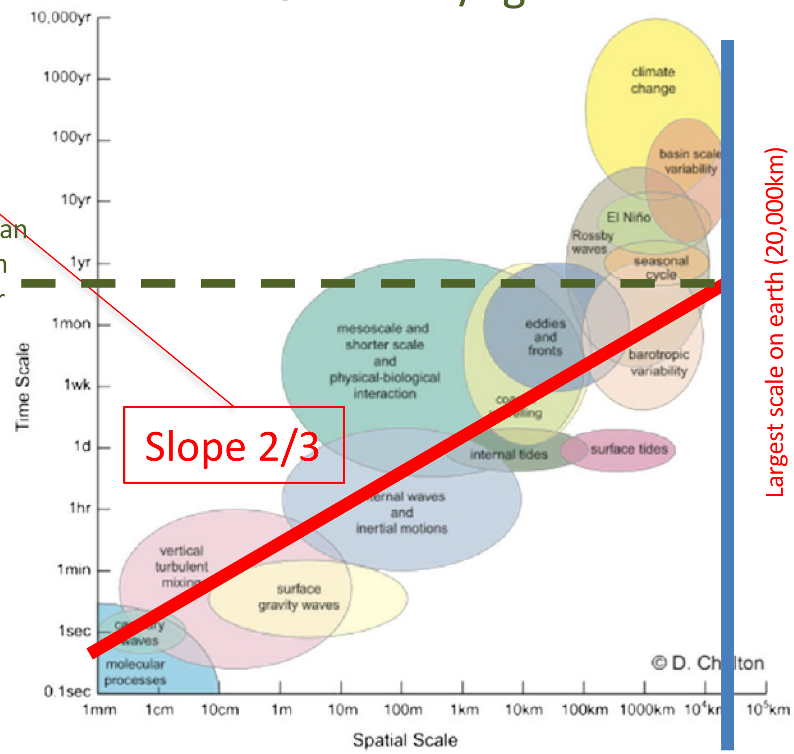
10 days: weather macroweather transition



Ocean

$\varepsilon \approx 10^{-8}$W/kg

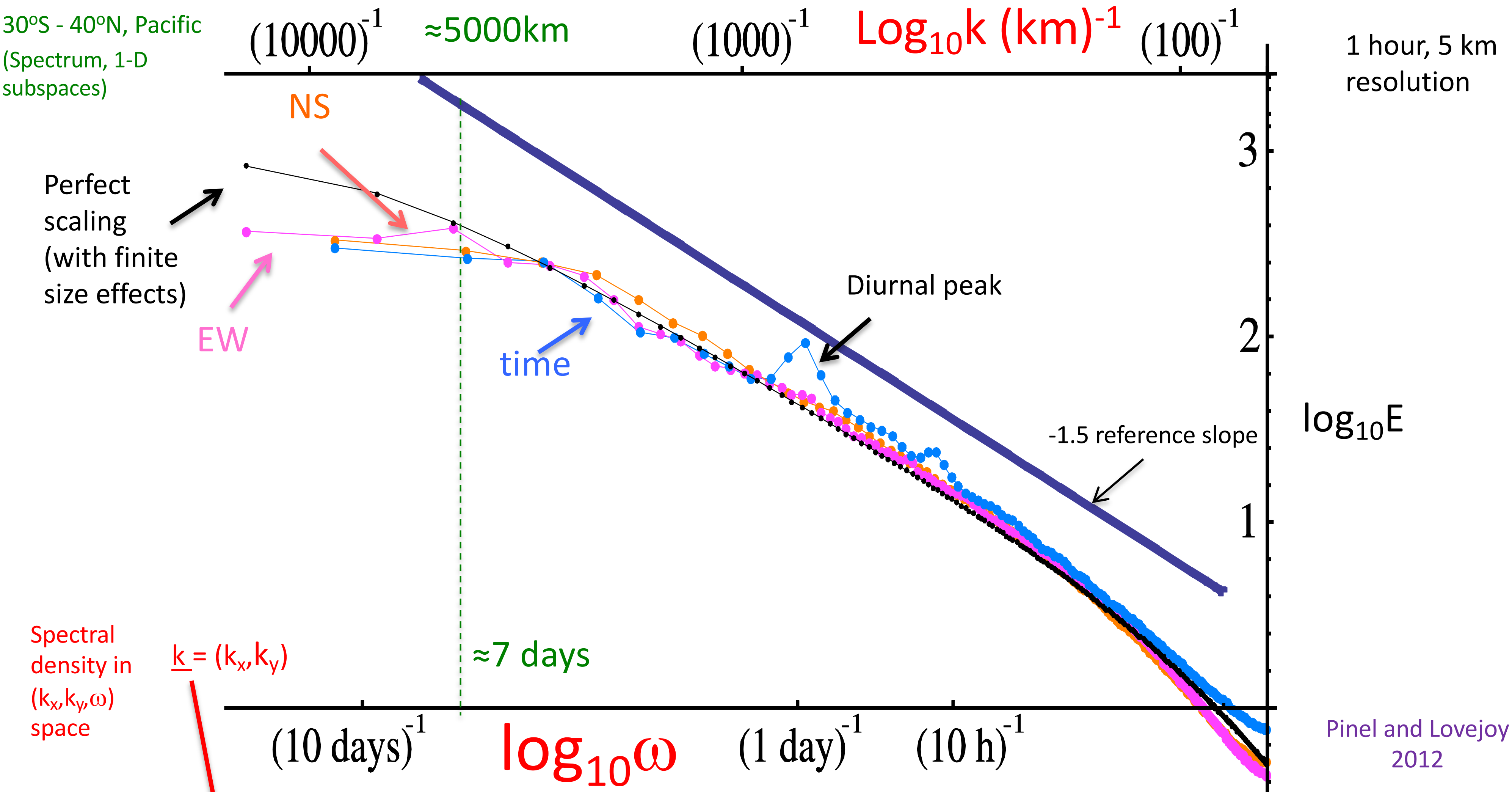
6 months: ocean weather ocean macroweather transition



Updated from Ghil, Lucarini 2020 (courtesy D. Chelton)

2012: Impact of data and scaling:

1400 Geostationary IR images ((x,y,t): 1000x1000x1400 pixels)



$$P\left(\lambda^{-1}\left(\underline{k},\omega\right)\right)=\lambda^{s}P\left(\left(\underline{k},\omega\right)\right)$$

Accurate space-time scaling

$$P\left(\omega,\underline{k}\right)\propto\left\langle\left|\tilde{I}\left(\omega,\underline{k}\right)\right|^{2}\right\rangle$$

Fluctuations, Wavelets

Fluctuation exponent

Mean fluctuation

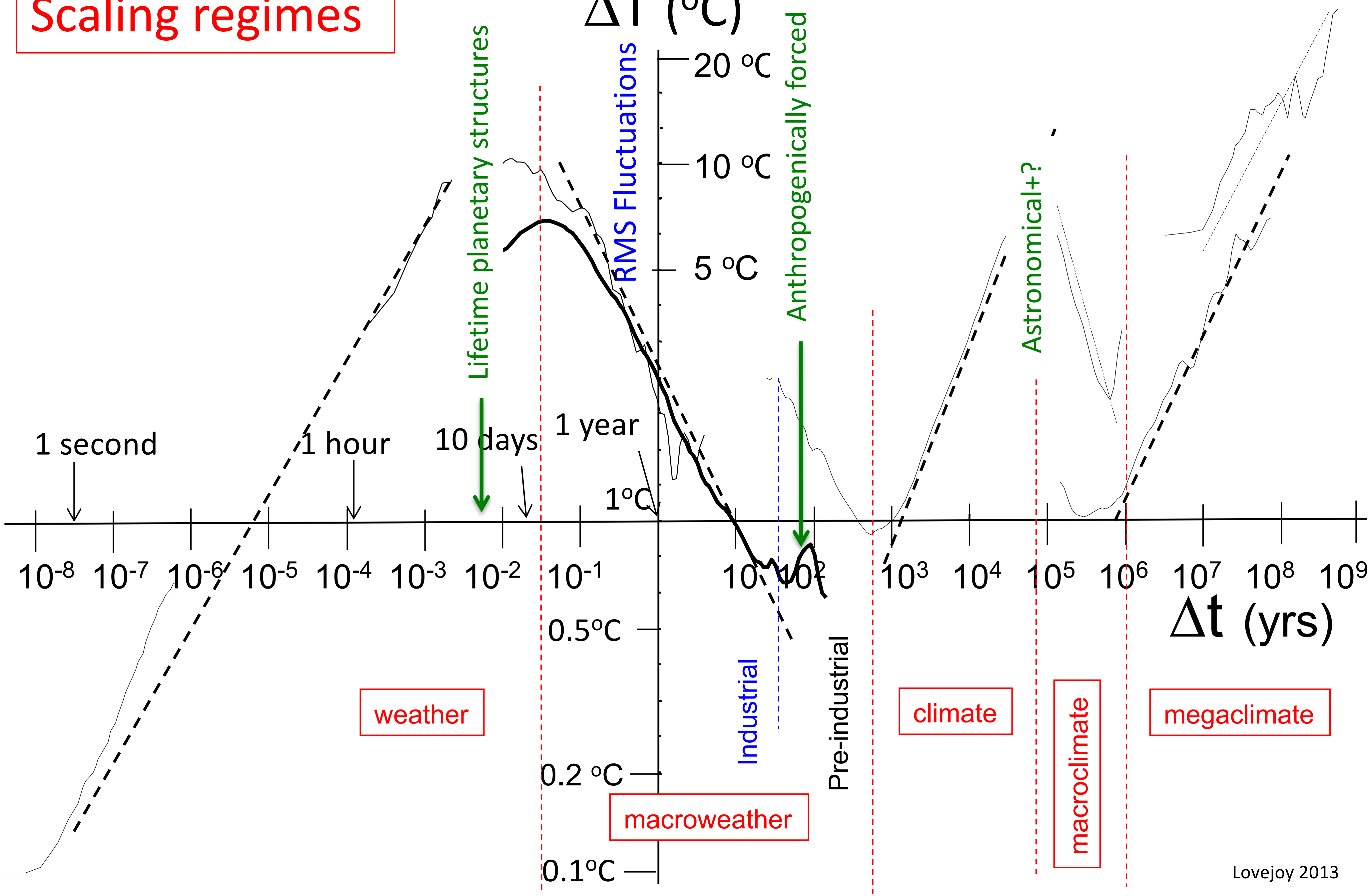
$$\left\langle \Delta T\left(\Delta t\right)\right\rangle \approx \Delta t^{H}$$

Fluctuation (defined by wavelets…)

New simple technique (re)discovered in 2012: Fluctuation analysis

Scaling regimes

Based on Haar wavelets (1910)



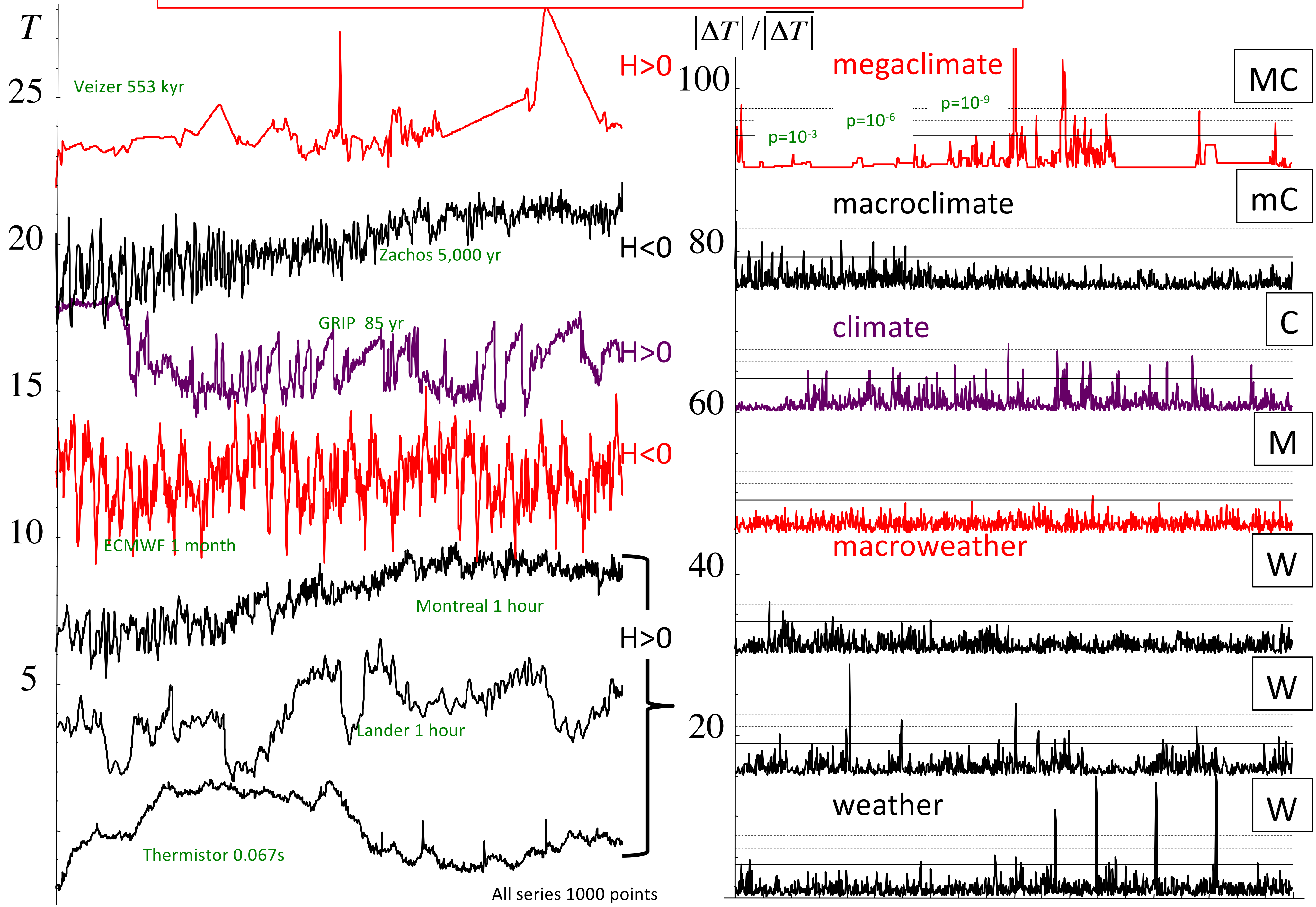
Lovejoy 2013

Multifractality 1:

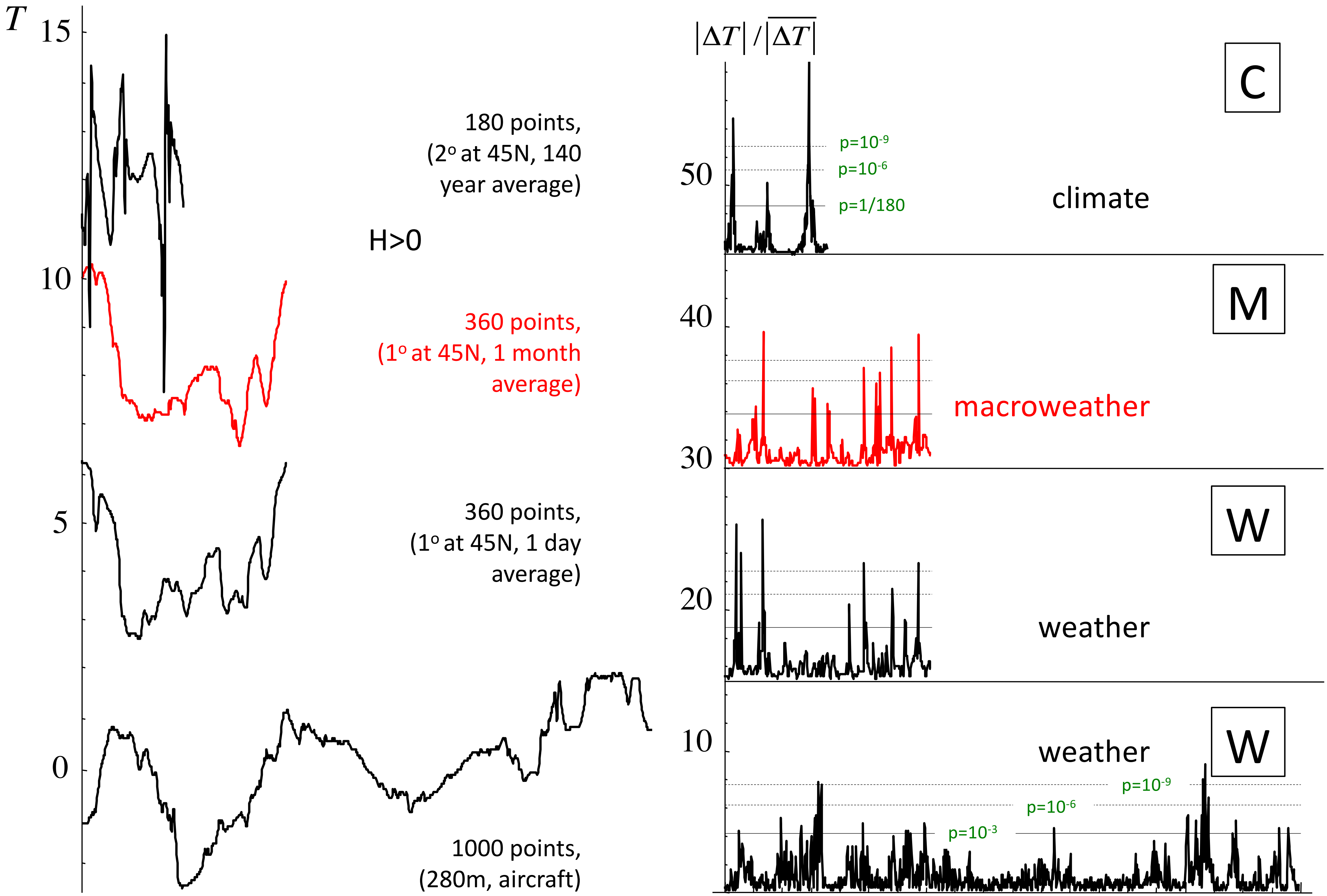
Intermittency

Spikes

Multifractality, Intermittency: Time



Multifractality, Intermittency: Space



Fluctuations and intermittency

Fluctuation → Driving flux

$$\Delta T\left(\Delta t\right) = \varphi_{\Delta t}\Delta t^{H}$$

$$\left\langle \Delta T\left(\Delta t\right)^{q}\right\rangle = \left\langle \varphi_{\lambda}^{q}\right\rangle \Delta t^{qH} \propto \Delta t^{-K(q)+qH}$$

Structure function

$$\xi\left(q\right) = qH - K\left(q\right)$$

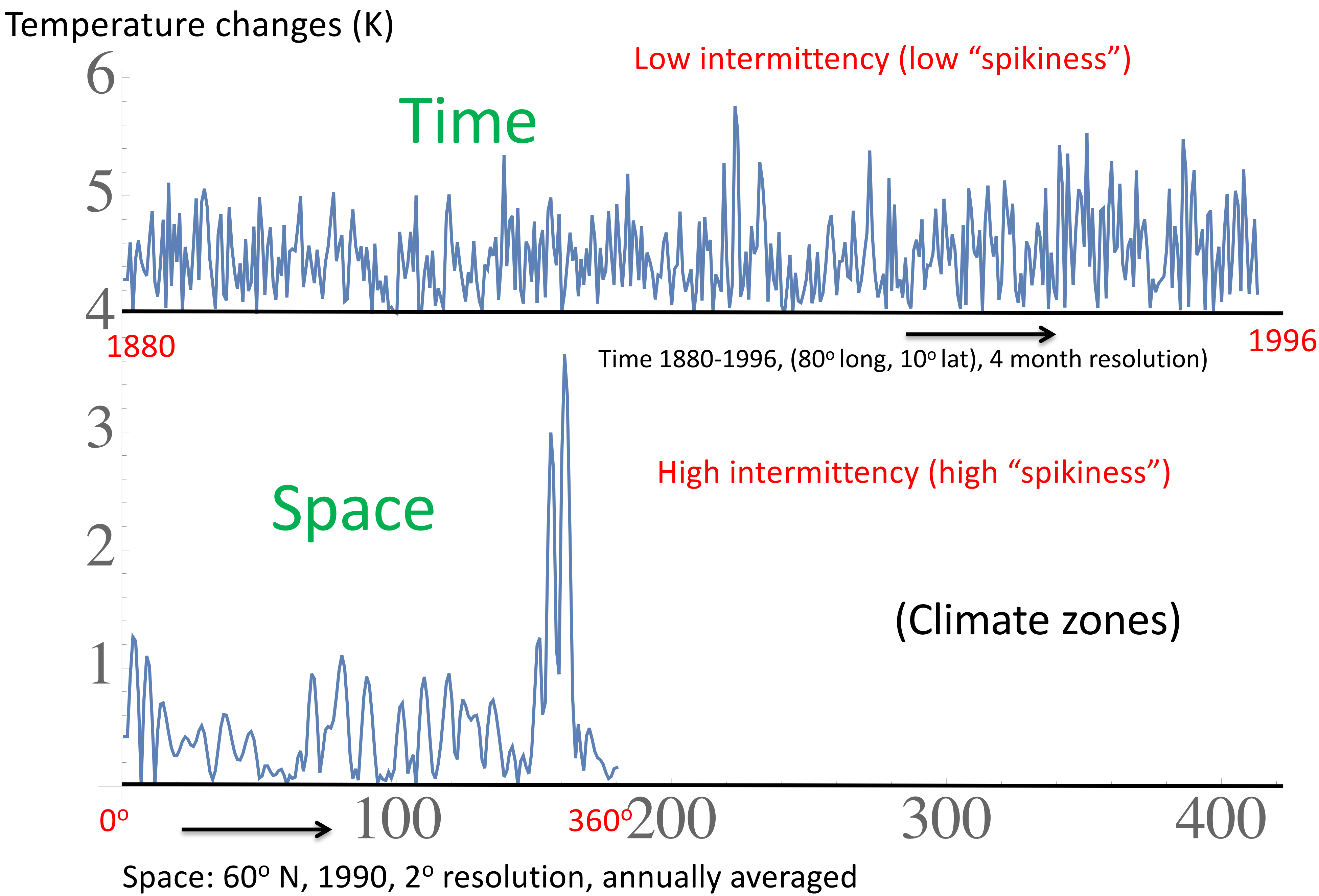
$$\left\langle \varphi_{\lambda}^{q}\right\rangle = \lambda^{K(q)}; \quad \lambda = \tau / \Delta t$$

Mean/RMS

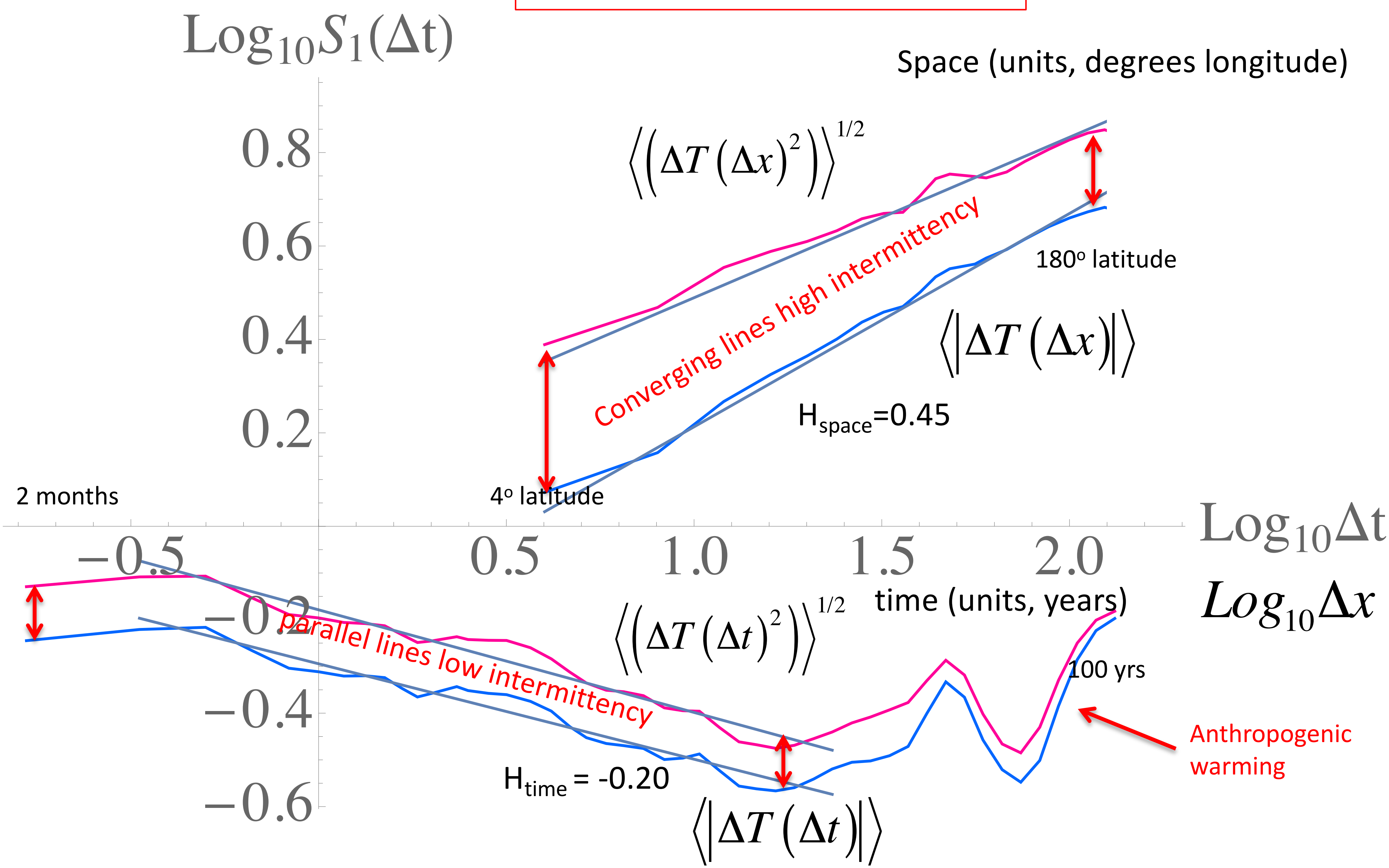
$$\left\langle \Delta T\left(\Delta t\right)\right\rangle / \left\langle \Delta T\left(\Delta t\right)^{2}\right\rangle^{1/2} \propto \Delta t^{K(2)/2} \approx \Delta t^{C_1}$$

=0 for Gaussian processes

Macroweather spatial and temporal intermittency



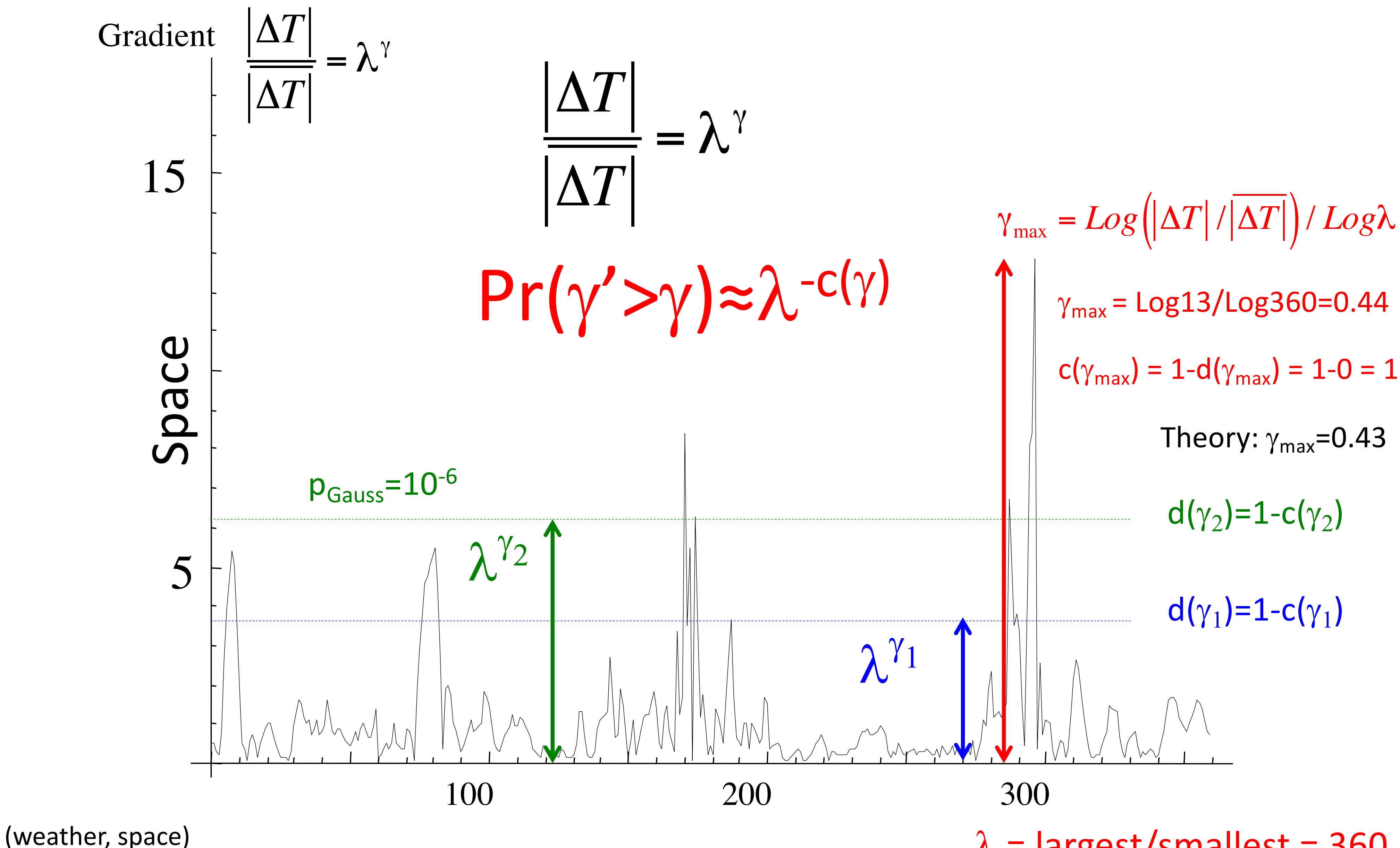
(Haar) Fluctuations



L 2022

top are spatial temperature fluctuations, bottom temporal at equator

Multifractal interpretation



Multifractality 2:

Cascades

Cascades

Generic statistical behaviour:

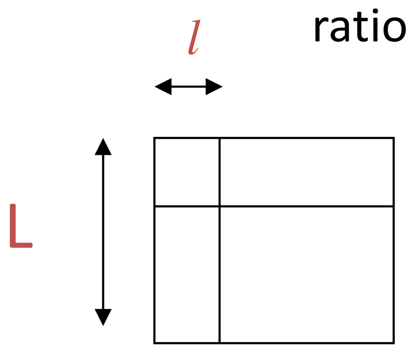
Scale invariant

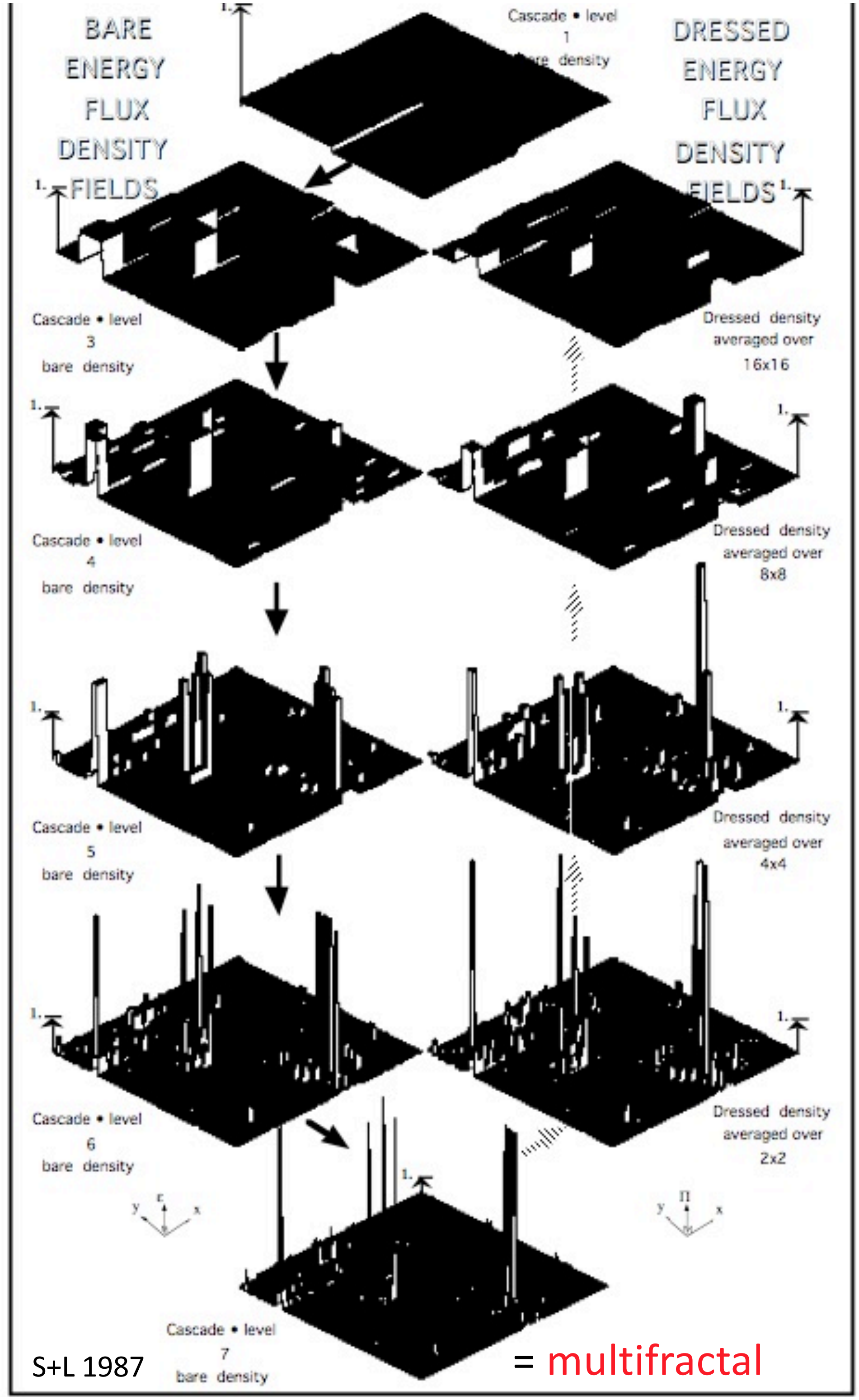
$$\left\langle \varepsilon_{\lambda}^{q} \right\rangle \approx \lambda^{K(q)}$$

scaling

Statistical averaging

Resolution: ratio $\lambda = L/l$





Intermittency, multifractality (sparse, "spikes") .....the exponents C_1 ,α

Statistics of the "spikes"

$$\frac{|\Delta T|}{\overline{|\Delta T|}} = \lambda^{\gamma} \qquad \left\langle \varepsilon_\lambda^q \right\rangle = \lambda^{K(q)}; \quad \Pr\left(\varepsilon > \lambda^{\gamma}\right) \approx \lambda^{-c(\gamma)}$$

(codimension multifractal formalism, S+L 1987)

$$c(\gamma) = \max_q \left(q\gamma - K(q)\right)$$

$$K(q) = \max_\gamma \left(q\gamma - c(\gamma)\right)$$

Legendre transformation
(Parisi and Frisch 1985)

Characterization near the mean: $C_1 = K'(1)$

α and Universal multifractals
(S+L 1987)

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^{\alpha} - q\right); \quad c(\gamma) = C_1\left(\frac{\gamma}{C_1\alpha'} + \frac{1}{\alpha}\right)^{\alpha'}$$

$$\frac{1}{\alpha} + \frac{1}{\alpha'} = 1$$

$0 \le \alpha \le 2$

Empirical analysis: Estimating fluxes from the fluctuations

Multifractal cascade equation: $\langle \varphi_\lambda^q \rangle = \lambda^{K(q)}$

Fluctuations: $\Delta T = \varphi_{\Delta t} \Delta t^H$

Estimating the fluxes from the fluctuations

$$\varphi'_\lambda = \frac{\varphi_\lambda}{\langle \varphi_\lambda \rangle} \approx \frac{\Delta T(\Delta t)}{\langle \Delta T(\Delta t) \rangle}; \quad \lambda = \frac{\tau}{\Delta t}$$

outer cascade scale

Normalized flux at resolution λ

The "spikes"

$$M_q = \langle \varphi_\lambda'^q \rangle$$

Estimate at finest resolution, then degrade to intermediate resolutions by averaging

"Trace moments"

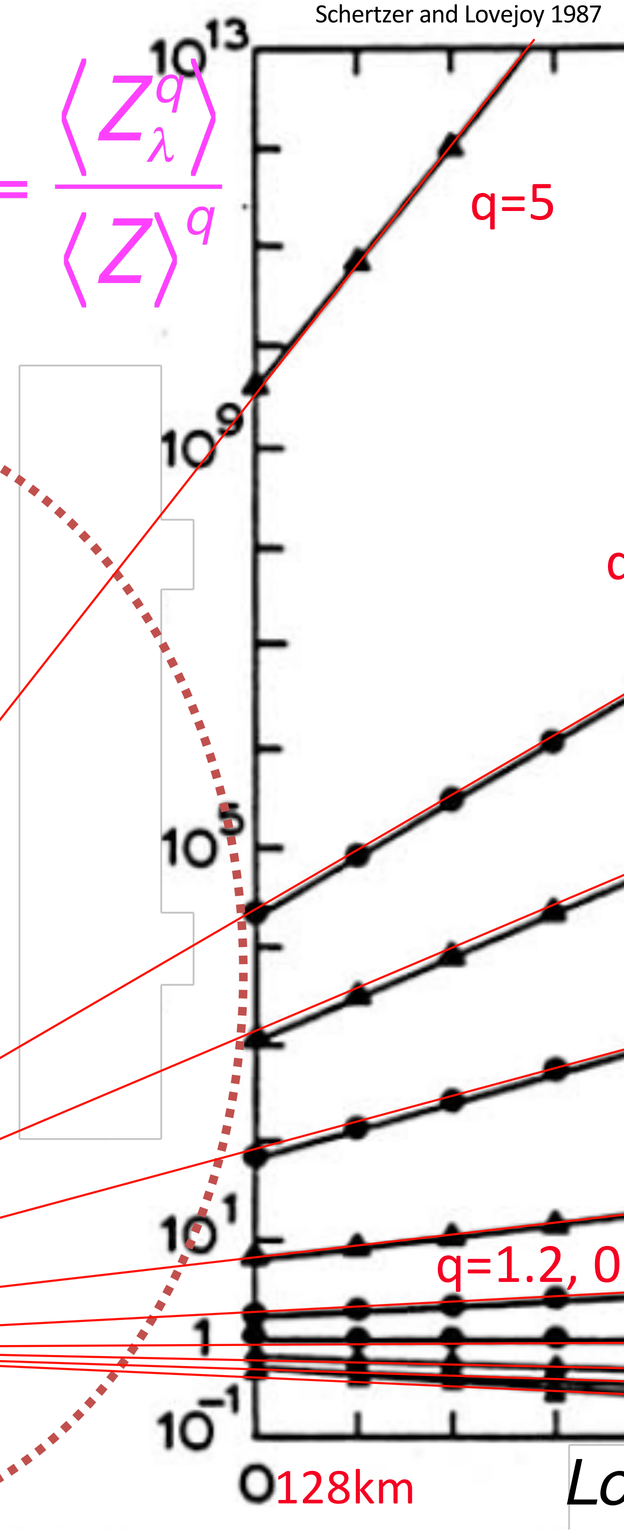
= The statistics of the spikes at different scales

Early evidence of cascades:

Precipitation 1987

(70 Radar Scans, Montreal, horizontal 3 weeks of rain data)

$$M = \frac{\langle Z_\lambda^q \rangle}{\langle Z \rangle^q}$$



Large scales

?

?

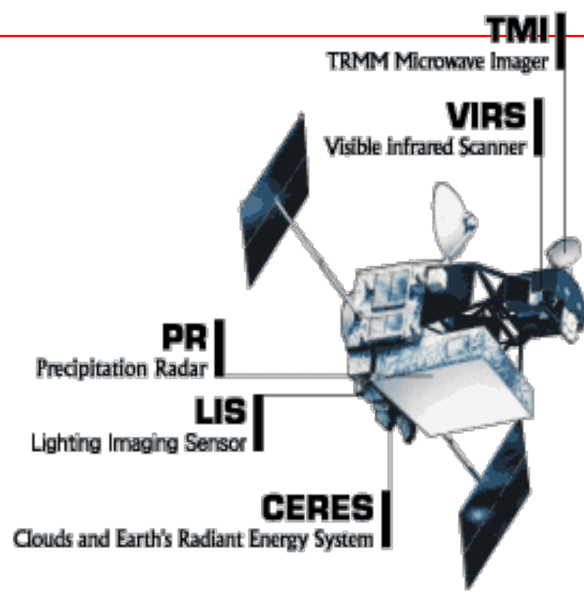
32,000km

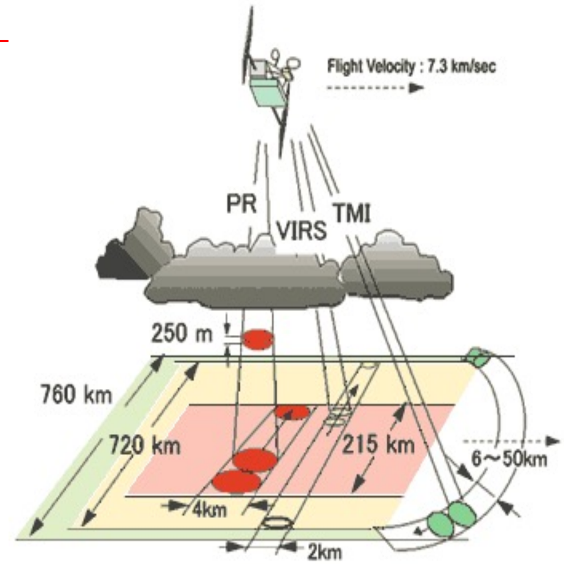
Cascade prediction:

$$\langle Z_\lambda^q \rangle / \langle Z_1 \rangle^q = \lambda^{K(q)}$$

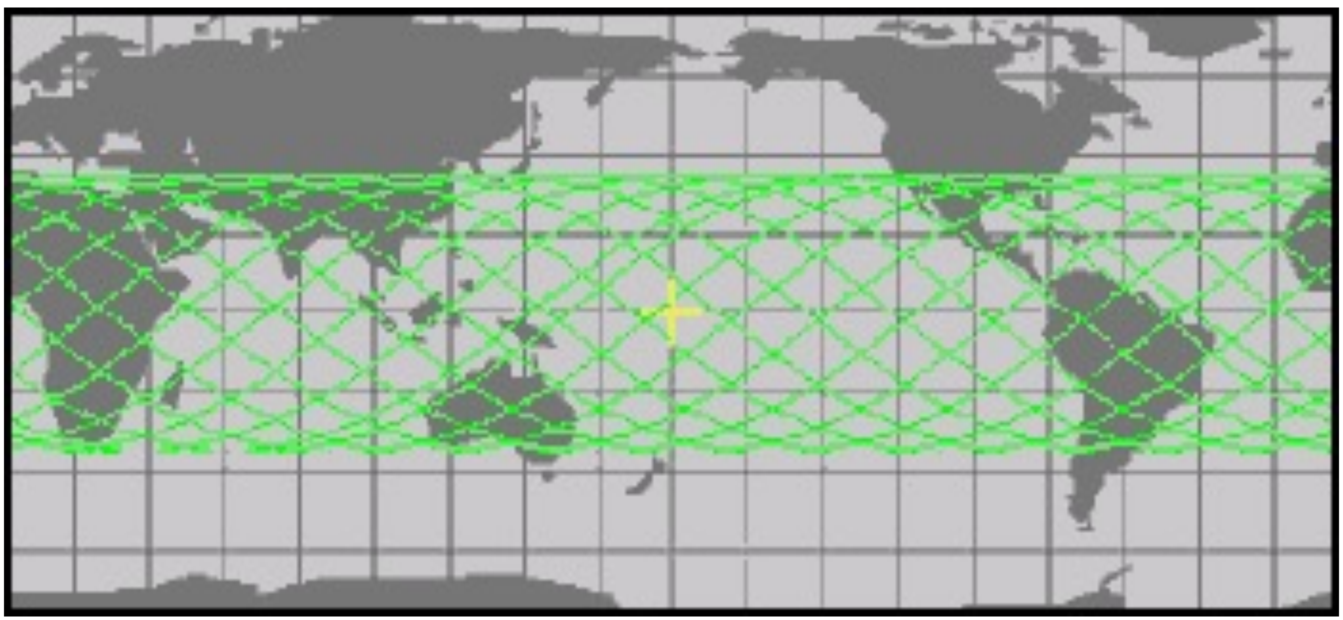
$$\lambda = L_{eff} / L_{res}$$

Tropical Rainfall Measuring Mission

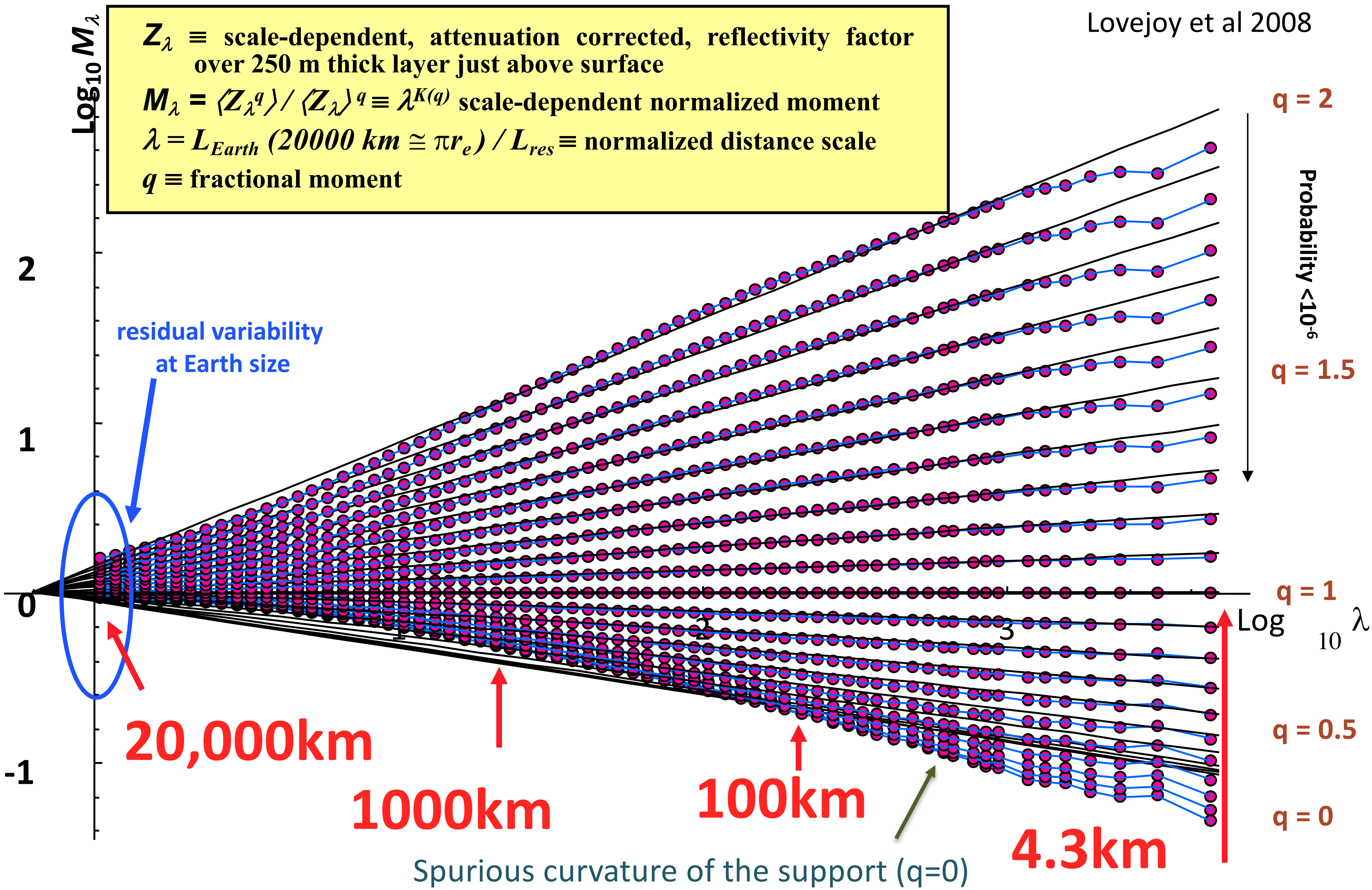




TRMM:
\$$10^9$ over
10 years



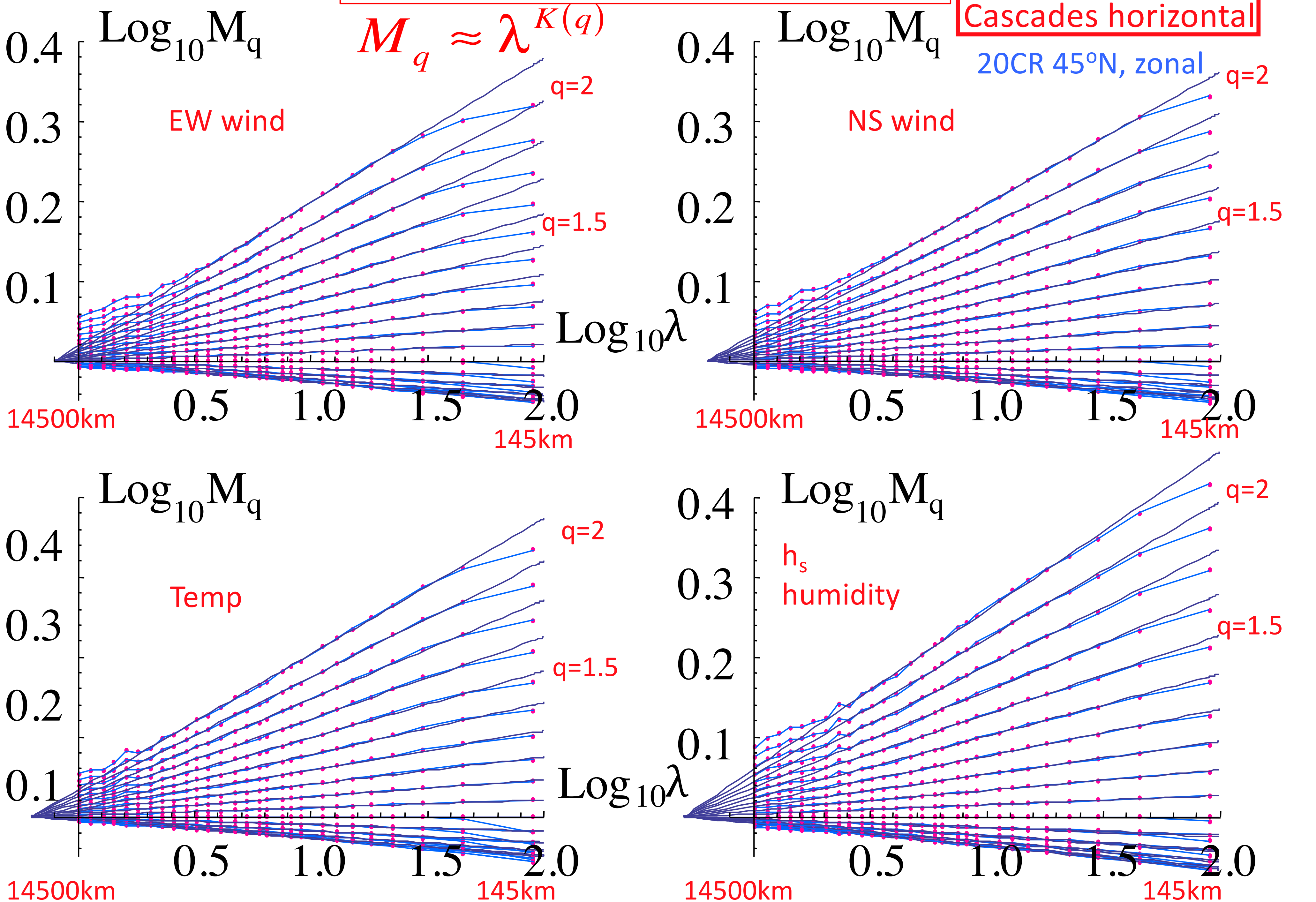
Scale-dependent TRMM PR Attenuation Corrected Reflectivity Factor [Z_λ] (1176 consecutive orbits -- ~70 days)



Reanalyses EW direction

$M_q \approx \lambda^{K(q)}$

Cascades horizontal



Trace moments: zonal analysis

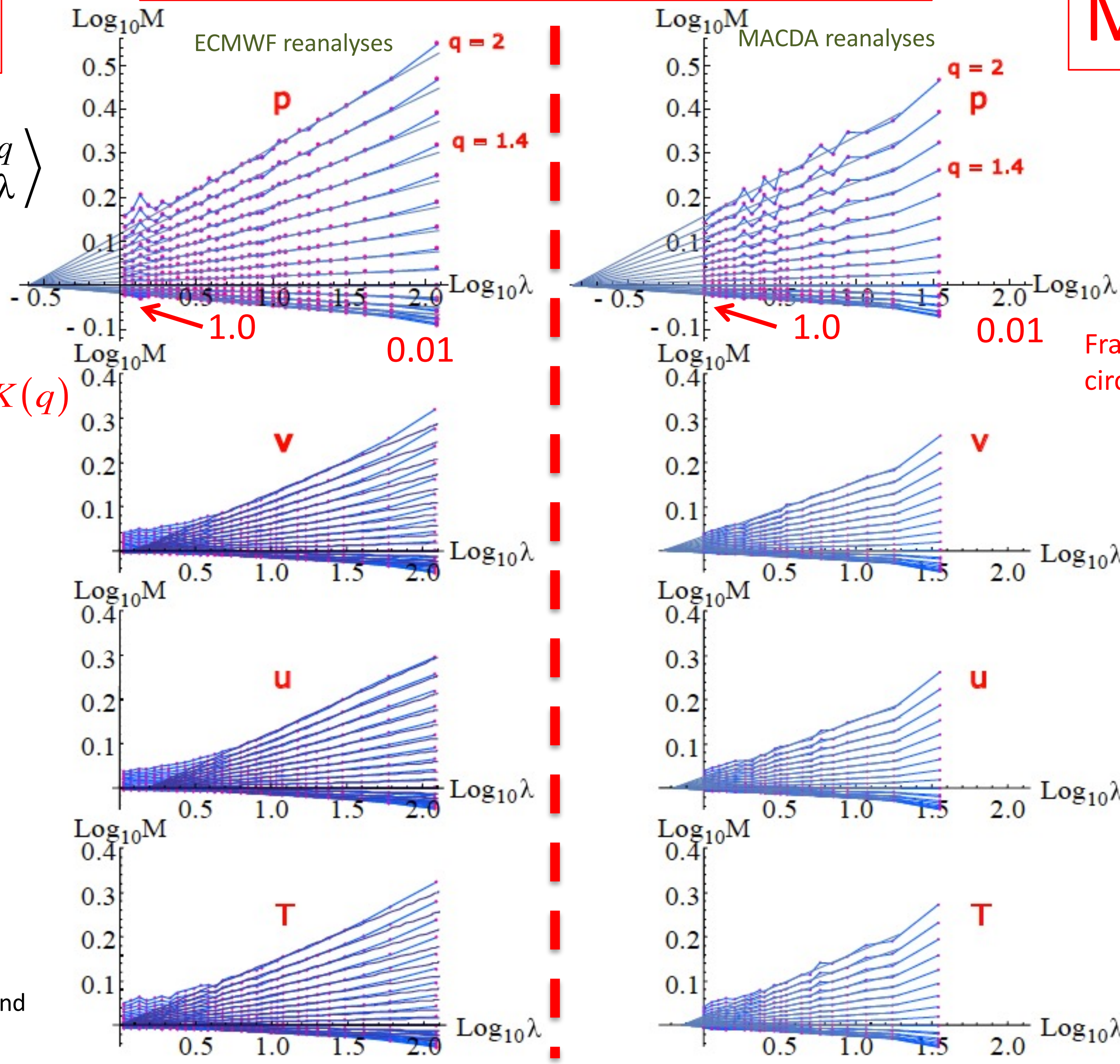
Earth | **Mars**

$$M_q = \langle \varepsilon_\lambda^q \rangle$$

Flux resolution λ

$$M_q \approx \lambda^{K(q)}$$

Predictions of cascade models



Chen, Lovejoy and Muller 2016

Earth versus Mars:
nearly identical multifractal cascades

Intermittency parameter

	Mars at 83% of Surface Pressure				Earth at 69% of Surface Pressure			
	U	V	T	p	U	V	T	p
C_1	0.078	0.075	0.089	0.119	0.084	0.087	0.077	0.119
α	1.97	1.98	1.92	2.17	1.85	1.85	1.90	2.01
L_{eff} Ratio	0.42	0.51	0.42	3.48	0.32	0.40	0.50	1.97

(EW analyses)

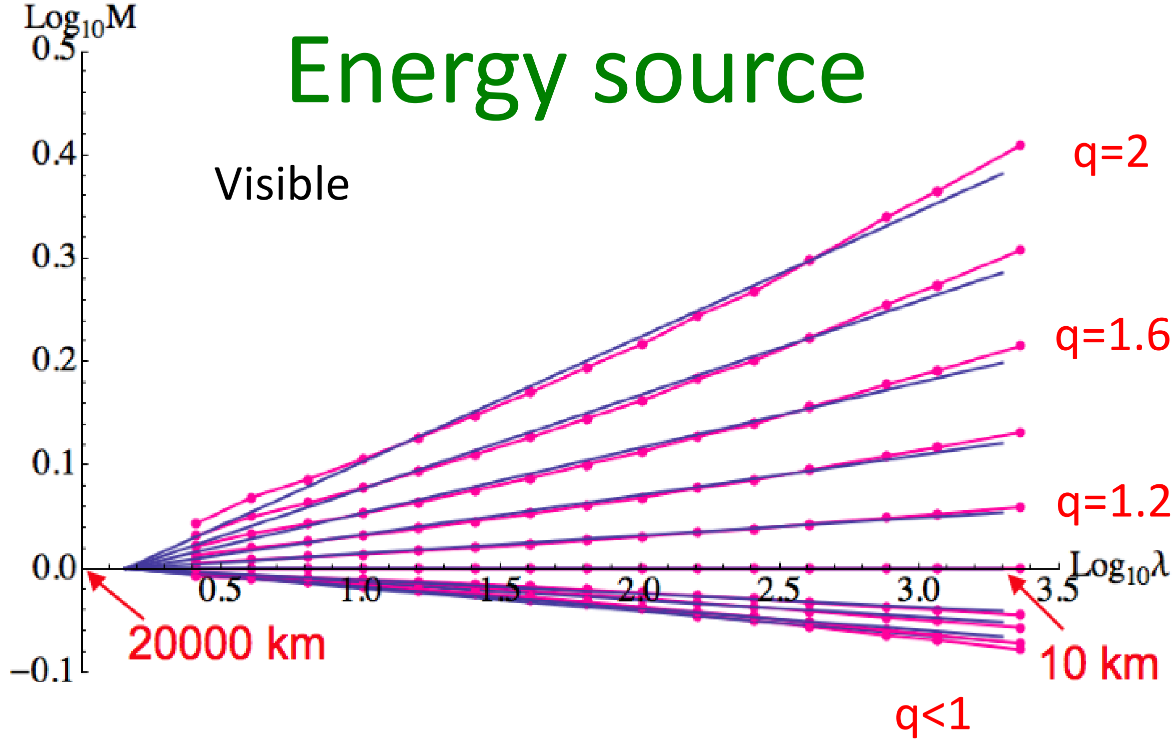
U = Zonal Wind, V = Meridional Wind, T = Temperature

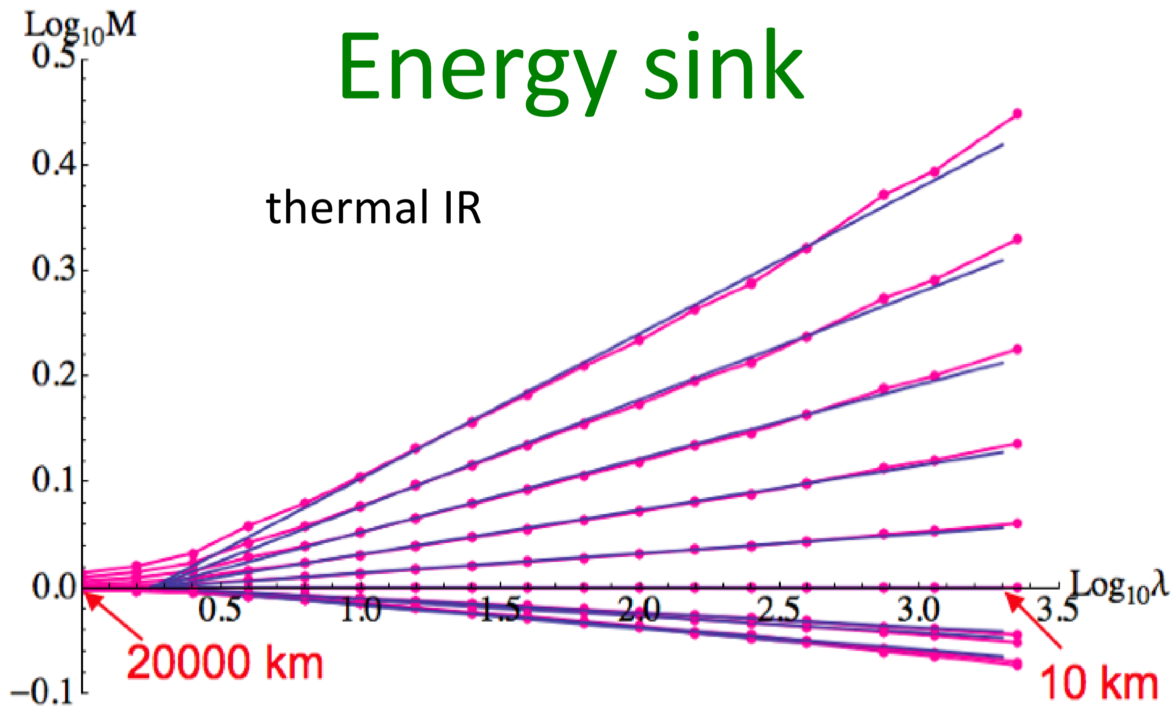
Nondimensional outer cascade scale

Data for Earth are from the ECMWF Reanalyses: Lovejoy and Schertzer (2011)

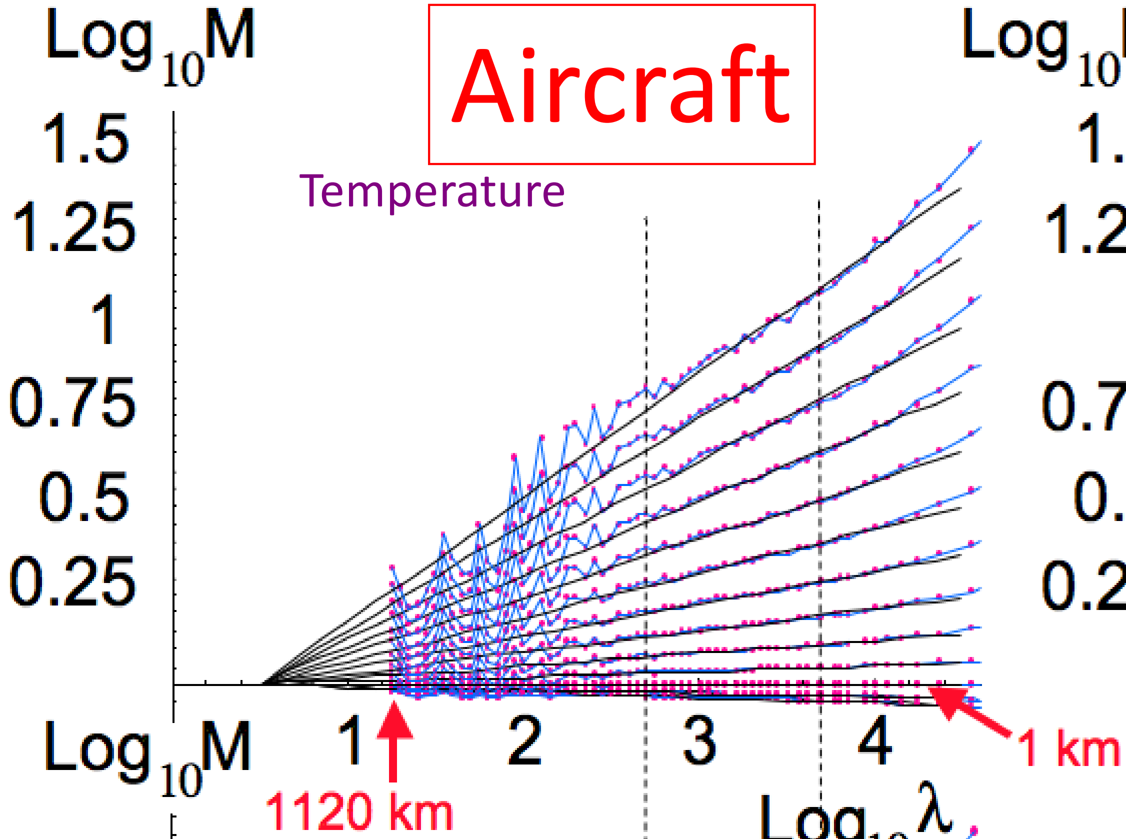
TRMM satellite data, ≈1000 orbits

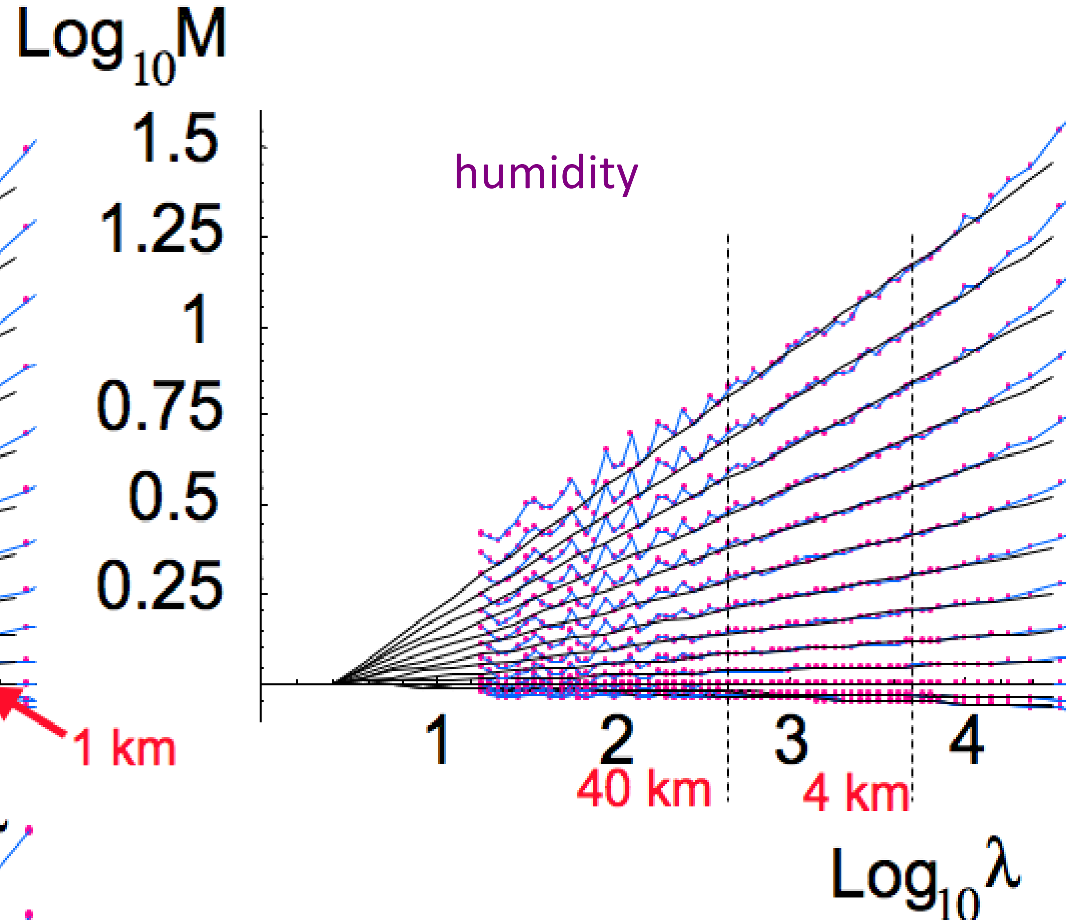
Energy budget

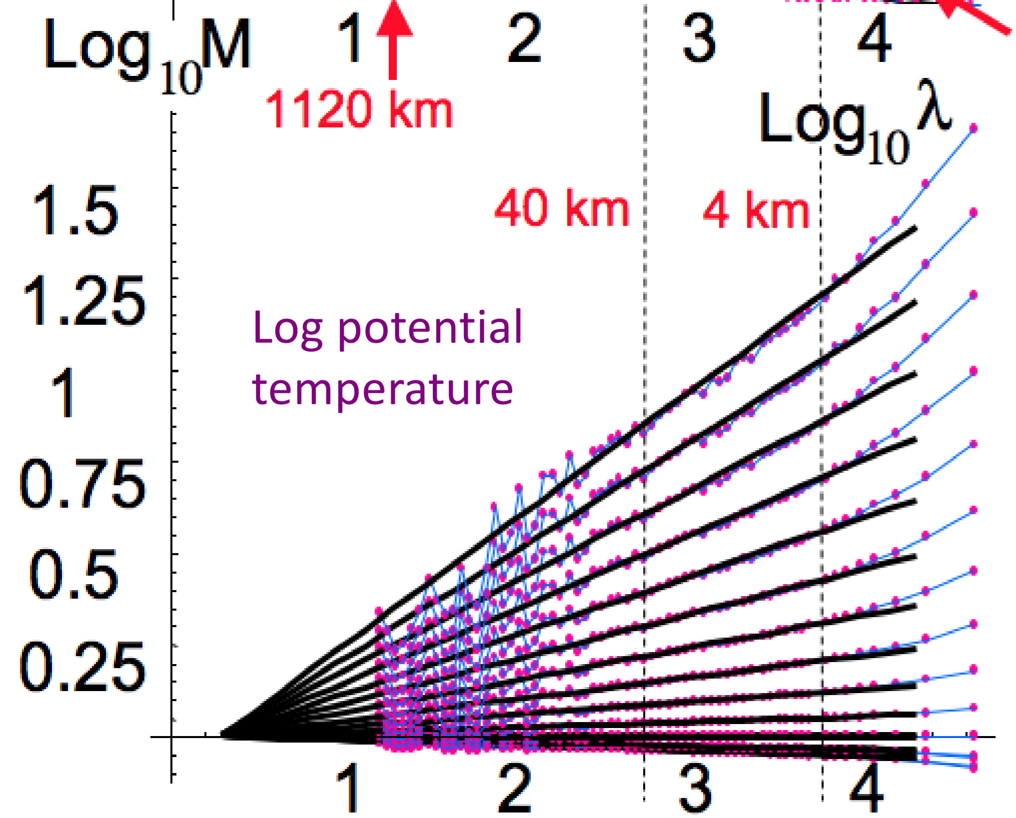




Aircraft







Horizontal cascades from 24 aircraft legs (11-13km)

Fields that are relatively unaffected by the trajectories

Governing Equations and Numerical Weather Prediction Models

Governing atmospheric Equations

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}.\,\mathrm{grad})\mathbf{u} - 2\boldsymbol{\Omega} \times \mathbf{u} - \alpha\,\mathrm{grad}\,p - \mathrm{grad}\,\Phi + \mathbf{F}$$ Conservation of momentum

$$c_v \frac{\partial T}{\partial t} = -c_v(\mathbf{u}.\,\mathrm{grad})T - \frac{p}{\rho}\,\mathrm{div}\,\mathbf{u} + Q$$ Conservation of energy

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u}.\,\mathrm{grad})\rho - \rho\,\mathrm{div}\,\mathbf{u}$$ Conservation of matter

$$p = \rho R T.$$ Equation of state

wind; Earth angular velocity; Gravitational potential; Specific volume=1/ρ; pressure; Friction; Specific heat; temperature; Heating rate; Gas constant; density

Important property: Scaling symmetry

Atmospheric laws → Anisotropic blowup → λ^{H} (Atmospheric laws)

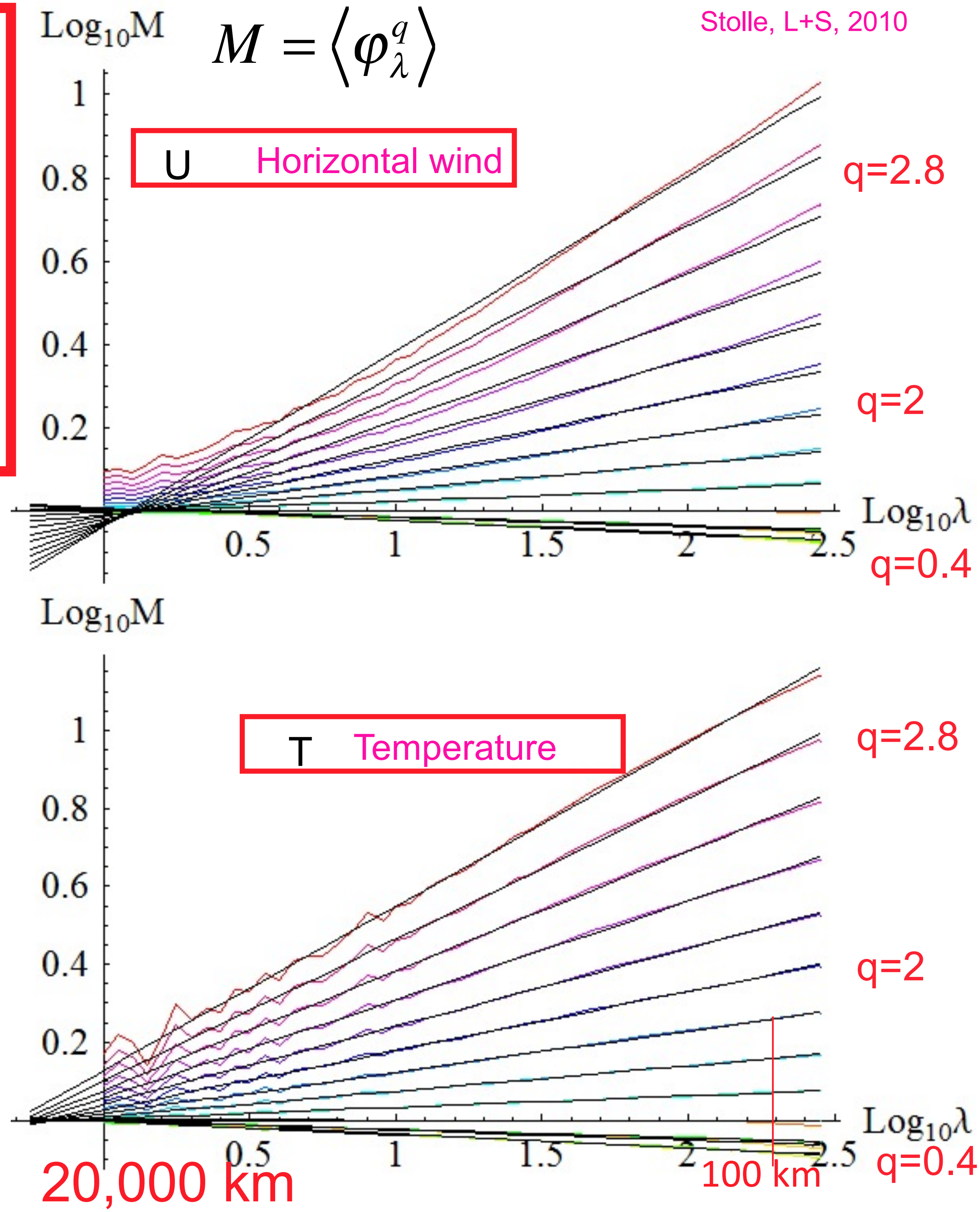
Factor λ

S+L 2012

Global GEMS Model 00h
(Numerical Weather Prediction Model)

Analysis of four months
U,T at 1000 mb

(48 h forecasts are
almost the same)



Summary

Horizontal spatial Scaling exponents

L+S 2013

		C_1	α	H	β	L_{eff}
State variables	*u, v*	0.09	1.9	1/3, (0.77)	1.6, (2.4)	(14 000)
	w	(0.12)	(1.9)	(−0.14)	(0.4)	(15 000)
	T	0.11, (0.08)	1.8	0.50, (0.77)	1.9, (2.4)	5000 (19 000)
	h	0.09	1.8	0.51	1.9	10 000
	z	(0.09)	(1.9)	(1.26)	(3.3)	(60 000)
Precipitation	*R*	0.4	1.5	0.00	0.2	32 000
Passive scalars	Aerosol concentration	0.08	1.8	0.33	1.6	25 000
Radiances	Infrared	0.08	1.5	0.3	1.5	15 000
	Visible	0.08	1.5	0.2	1.5	10 000
	Passive microwave	0.1–0.26	1.5	0.25–0.5	1.3–1.6	5000–15 000
Topography	Altitude	0.12	1.8	0.7	2.1	20 000
Sea surface temperature	SST (see Table 8.2)	0.12	1.9	0.50	1.8	16 000

α: 1.5 – 1.9

C_1: range 0.08 - 0.12… except precipitation!

L_{eff}: outer scale ≈ 20,000km

$$\Delta I = \varphi \Delta x^{H} \quad \langle \varphi_\lambda^q \rangle = \lambda^{K(q)} \quad \lambda = L_{eff} / \Delta x \quad K(q) = \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right) \quad E(k) \approx k^{-\beta}$$

Empirical Conclusions of planetary scale horizontal analyses

1) Multifractal scaling (including spectra) with outer scale close to the scale of the planet is respected to within ±1% to ±2% for moments up to order q=2, up to 5000km.

2) It is accurately respected by

a) remotely sensed radiances
b) Reanalyses
c) Aircraft data
d) Numerical weather prediction models
e) On Mars: (same multifractal exponents)

How is this possible?!

Which symmetry is primary: isotropy or scaling?

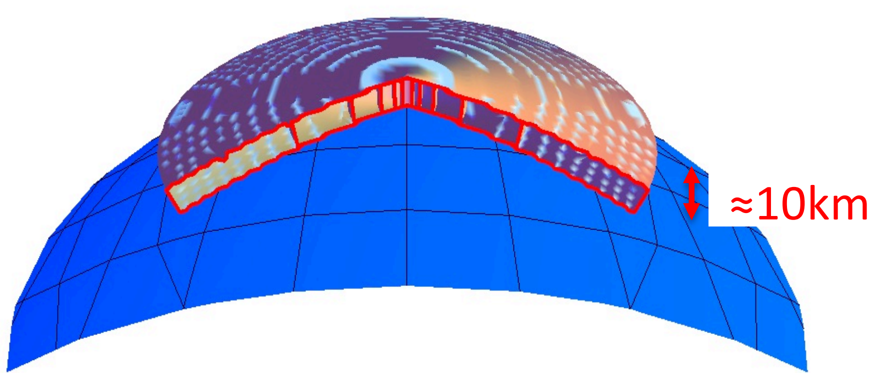
Isotropy first – then scaling

Motivation:

-the simplicity of isotropic theory
-theoretical approximations that confine the effects of gravity (buoyancy) to small scales

2-D isotropic
("Quasi-geostrophic")
turbulence

$D_{el}=2$



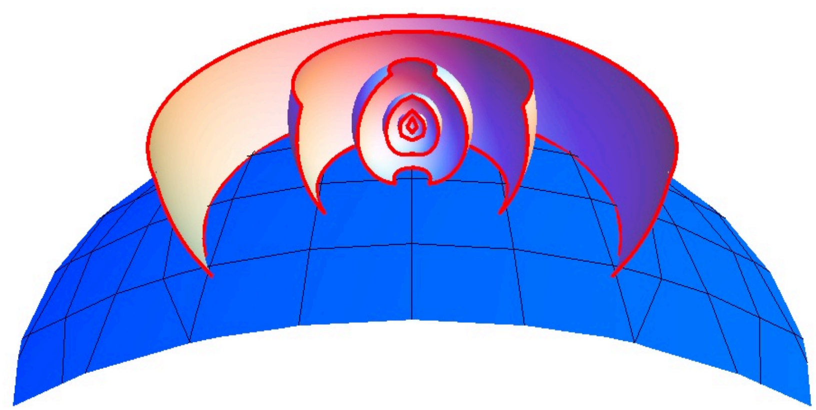
Only Scaling (anisotropic)

Motivation:

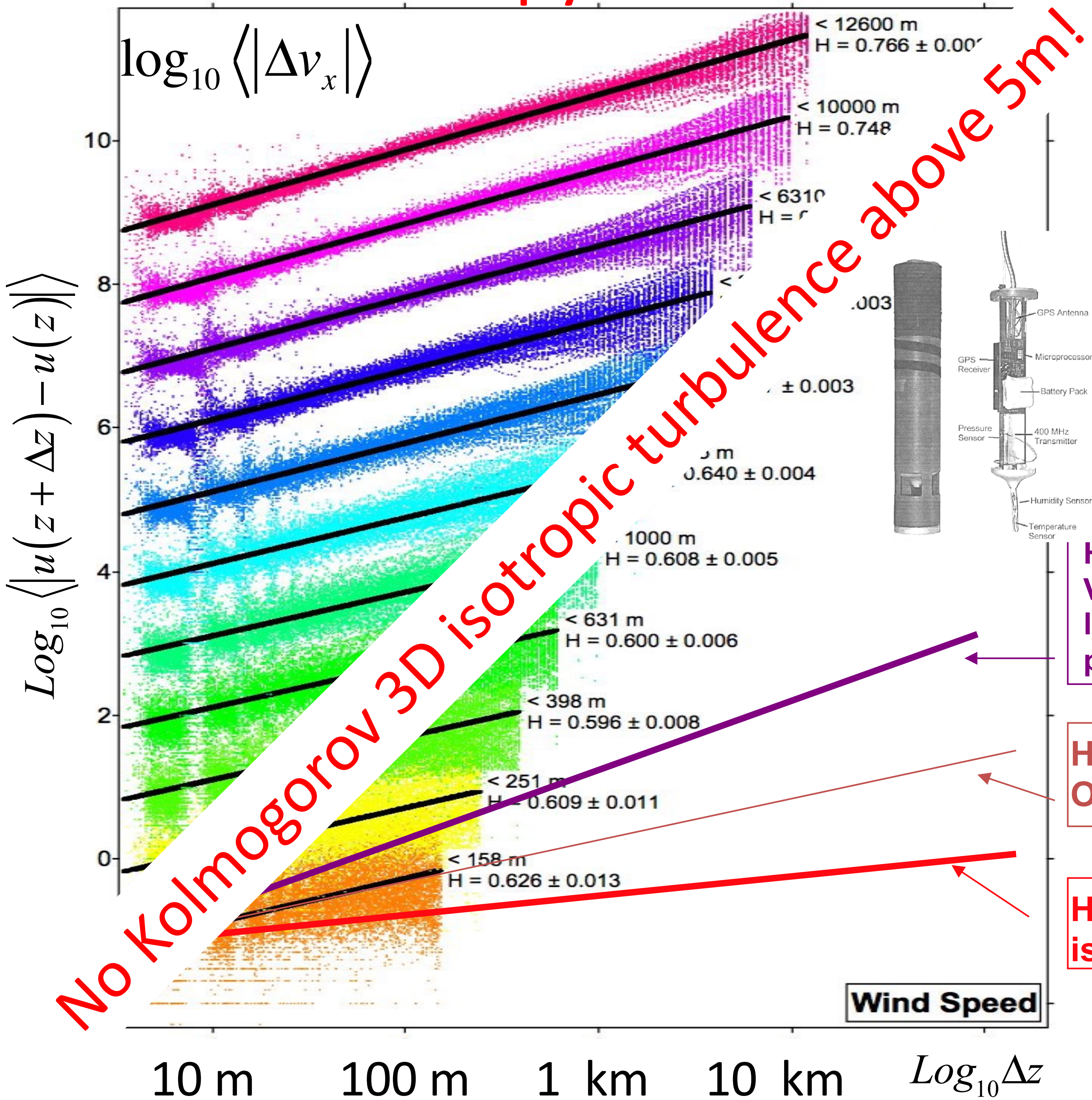
-Scaling of the governing equations
-gravity acts at all scales
-numerical models, data

3D isotropic turbulence

$D_{el}=3$



Anisotropy: the vertical



Velocity structure functions 237 drop sondes

H=1: Constant Brunt-Vaisaila frequency, quasi-linear gravity waves, or pseudo potential vorticity

H=3/5: Bolgiano-Obukhov value

H=1/3: Kolmorogov, 3D isotropic turbulence

Lovejoy, S., S. Hovde, A. Tuck, D. Schertzer, 2007

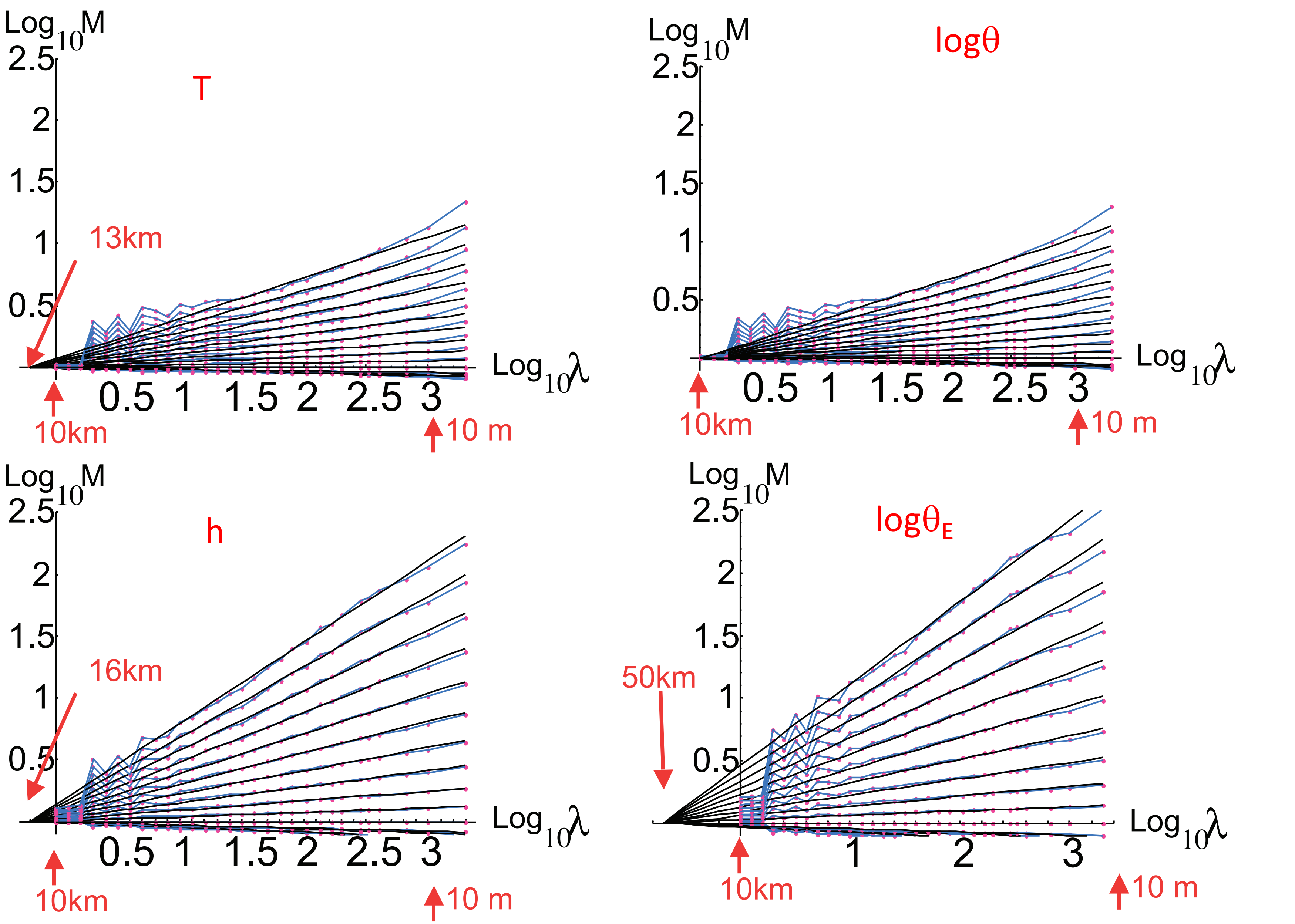
L, Tuck, Hovde, S, 2009

Vertical cascades:

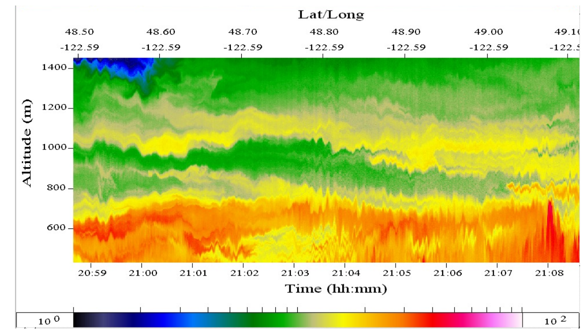
Thermodynamic fields (Dropsonde data)

$M = \langle \varphi_\lambda^q \rangle / \langle \varphi \rangle^q$

$M_q \approx \lambda^{K(q)}$



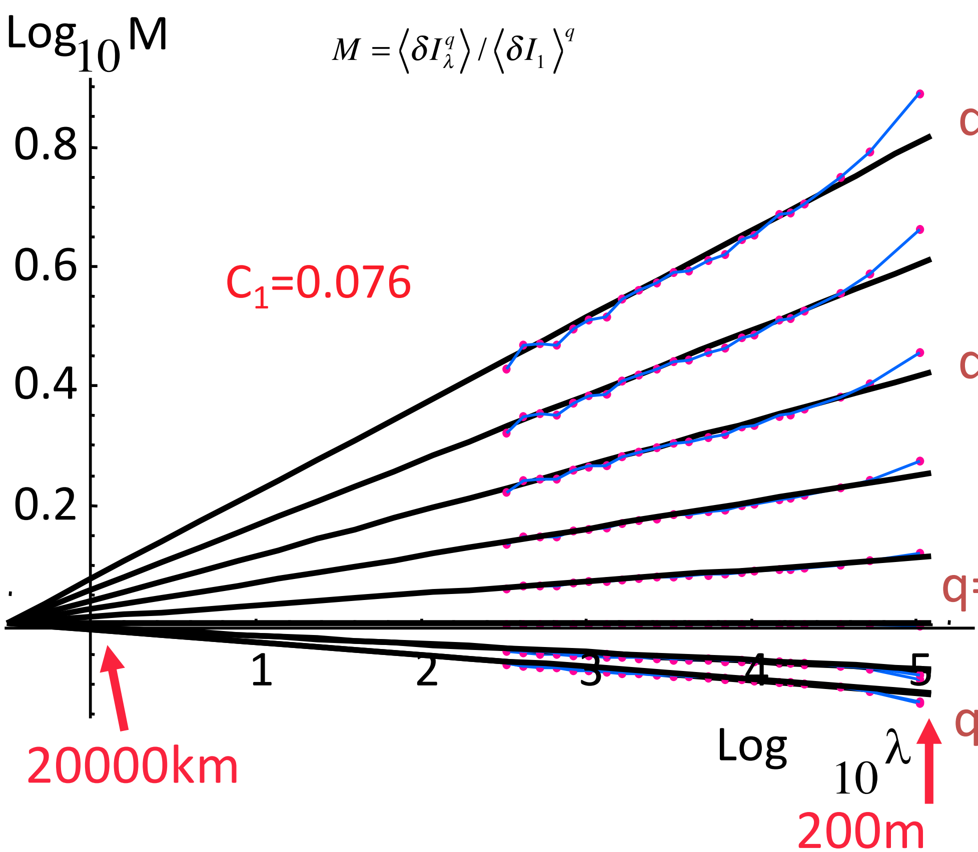
Vertical cascades:
lidar backscatter

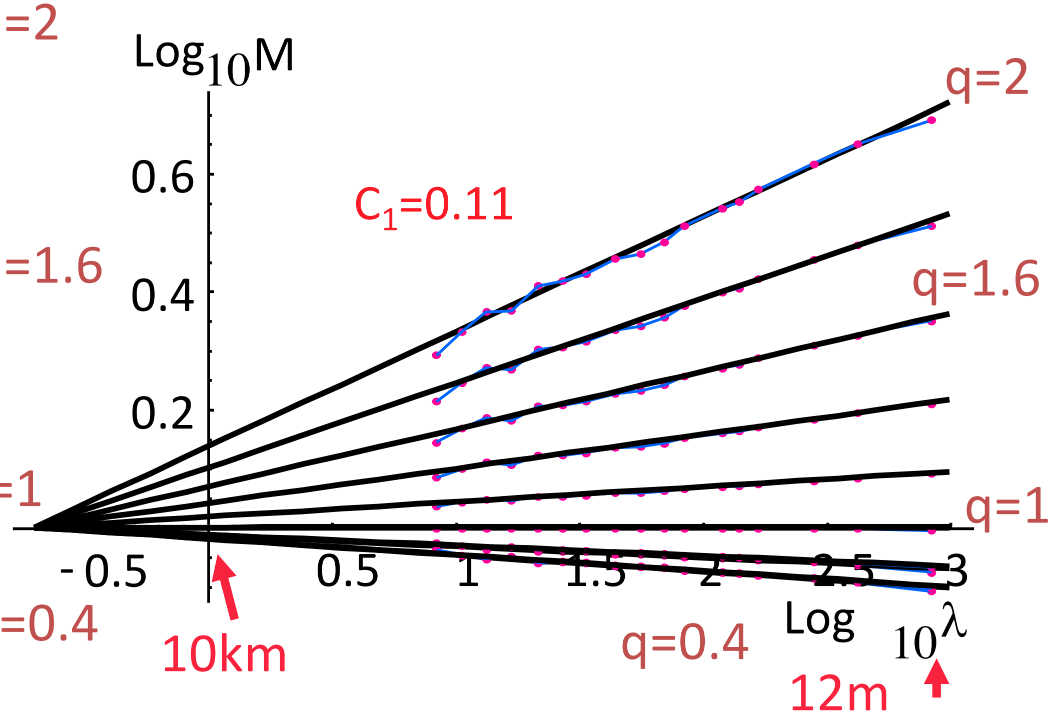


From 10 airborne lidar cross-sections near Vancouver B.C.

Horizontal cascade

Vertical cascade





q=0, 0.2, 0.4..., 2

L, Tuck, Hovde, S, 2009

The physical scale function and differential scaling

$$|\underline{\Delta r}| \rightarrow \|\underline{\Delta r}\|$$

Usual distance (=vector norm)

Scale function (scale notion)

Scale symmetry $\|\lambda^{-G}\underline{r}\| = \lambda^{-1}\|\underline{r}\|$

"canonical" scale function:

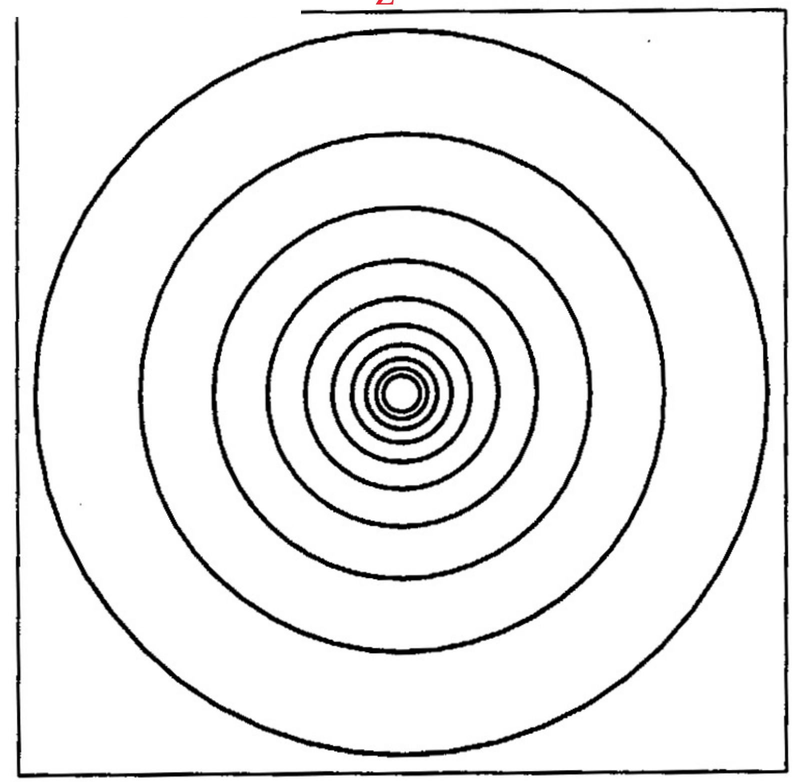
$$\|(\Delta x, \Delta z)\| = l_s\left(\left(\frac{\Delta x}{l_s}\right)^2 + \left(\frac{\Delta z}{l_s}\right)^{2/H_z}\right)^{1/2}$$

$$G = \begin{pmatrix} 1 & 0 \\ 0 & H_z \end{pmatrix}$$

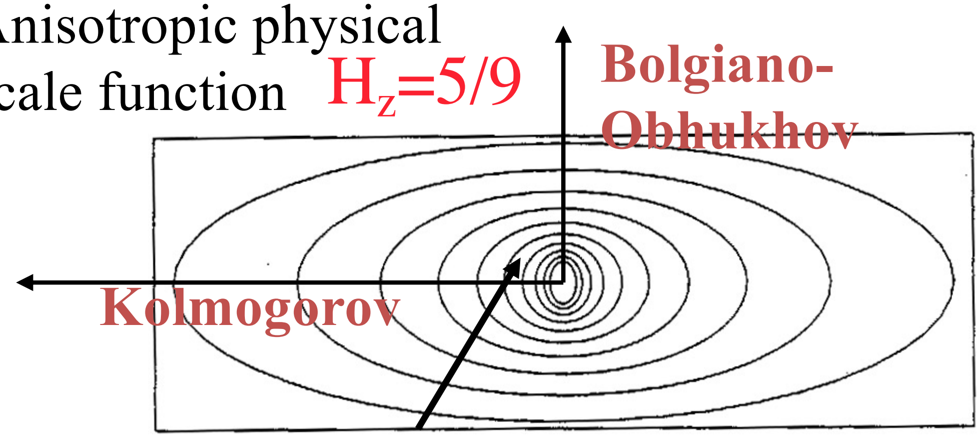
S+L 1985

Vertical sections

Isotropic function $H_z=1$

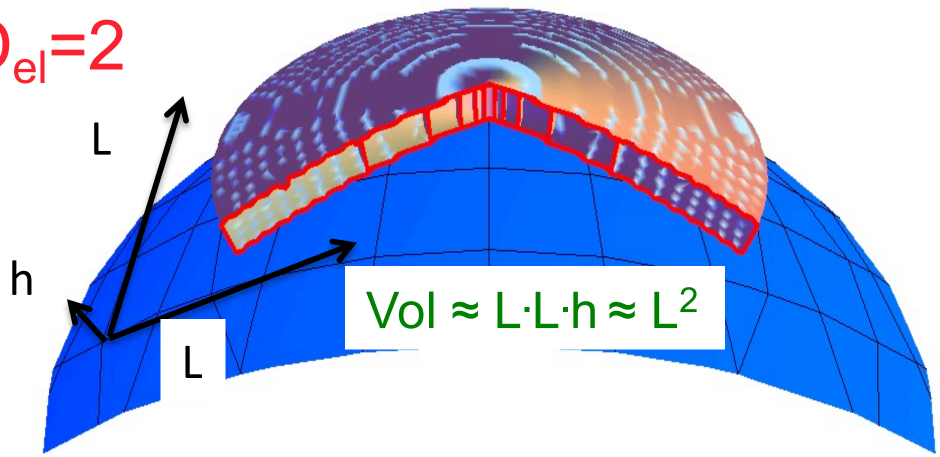


Anisotropic physical scale function $H_z=5/9$

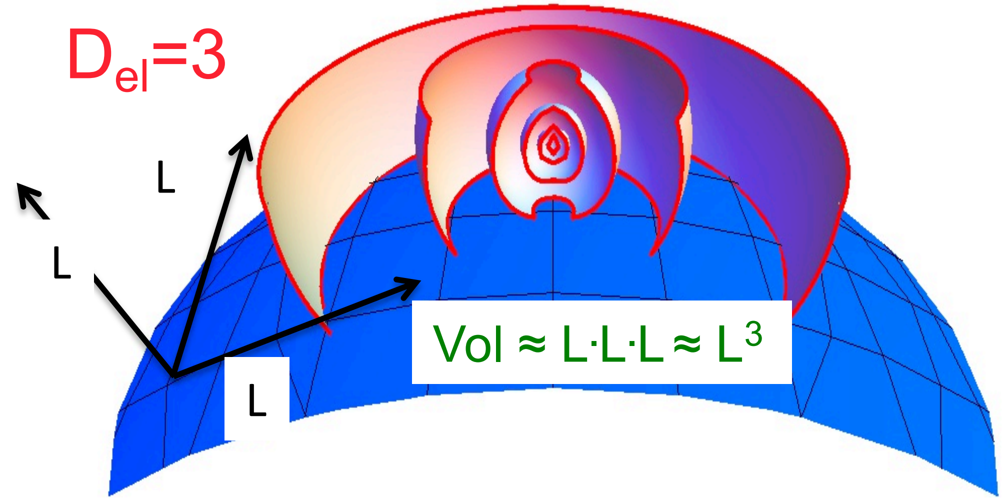


The 23/9D model

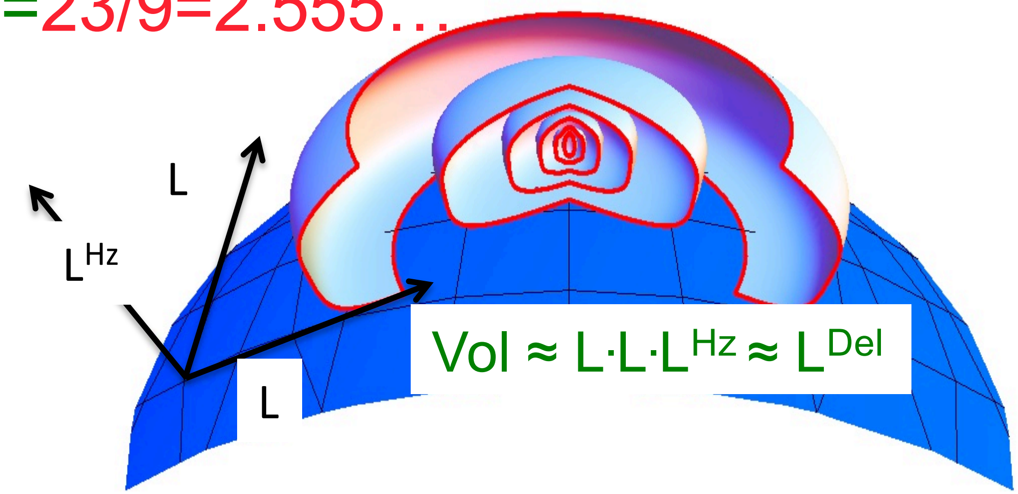
$D_{el}=2$



$D_{el}=3$



$D_{el}=1+1+H_z=23/9=2.555\ldots$



The 23/9D dynamics:

Bolgiano-Obukhov

$$\underbrace{\Delta v(\Delta x)=\varepsilon^{1/3}\Delta x}_{\text{Kolmogorov}}{}^{1/3};\quad \overbrace{\Delta v(\Delta z)=\phi^{1/5}\Delta z}^{\text{Bolgiano-Obukhov}}{}^{3/5}$$

Kolmogorov

$H_z=(1/3)/(3/5)=5/9$

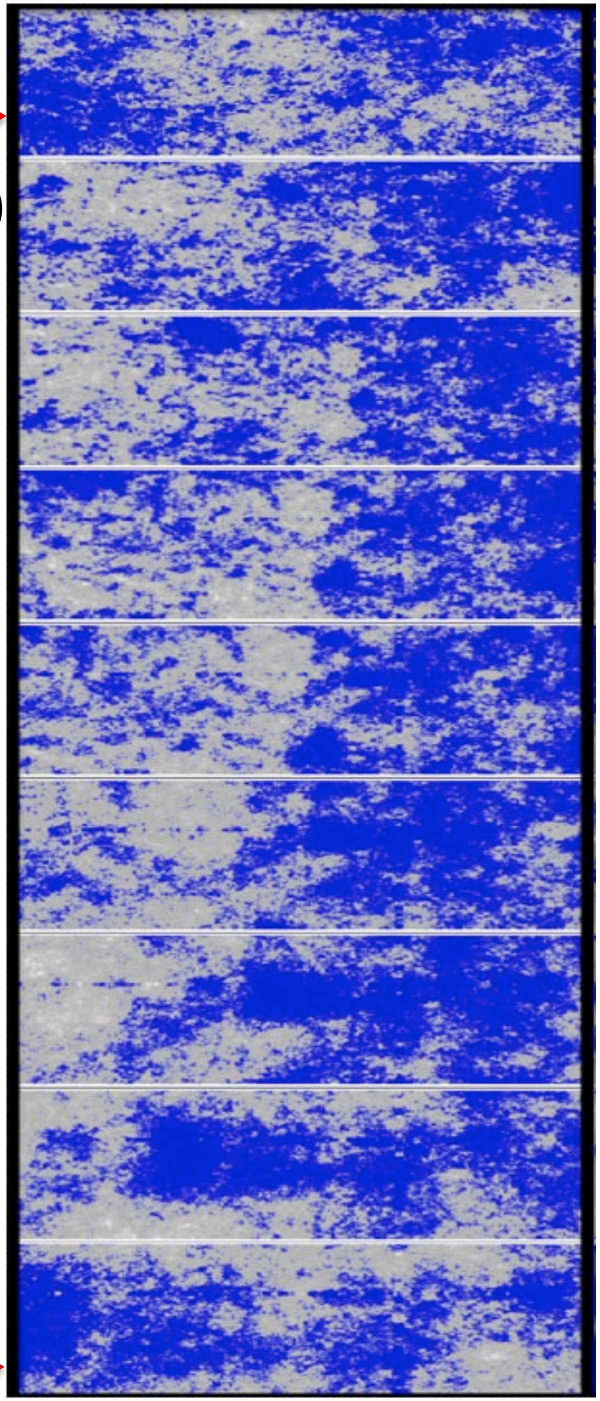
Anisotropic, Stratified Scaling

Stochastic

5km

Blow up X 2.9 (each)

Isotropic



Total: X5000

1 m

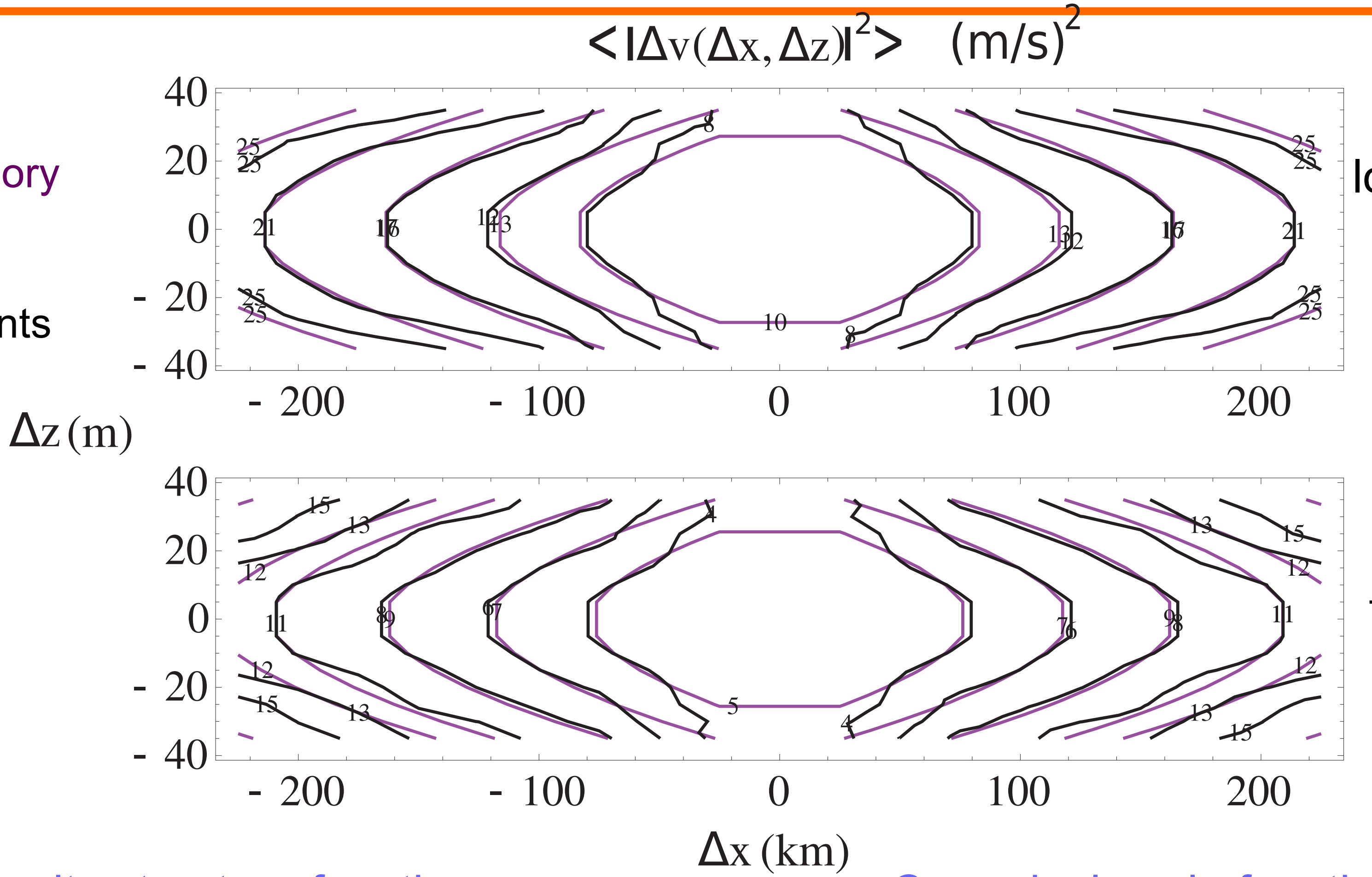
$H_z = 1$

14500 aircraft flights: 5-5.5km altitude, 2009, US (TAMDAR data)

Pinel L+S, 2012

Purple = theory

Black= measurements



Velocity structure function

$$\left\langle \Delta v^2 \left(\Delta x, \Delta z\right)\right\rangle = C\left\|\left(\Delta x, \Delta z\right)\right\|^{\xi(2)}$$

$$\xi(2) \approx 0.80$$

Canonical scale function

$$\left\|\left(\Delta x, \Delta z\right)\right\| = \left(\left(\frac{\Delta x}{l_s}\right)^2 + \left(\frac{\Delta z}{l_s}\right)^{2/H_z}\right)^{1/2}$$

$$H_z \approx 0.57 \pm 0.01$$

(Theory: 5/9=0.555…)

Empirical estimates of H_z:

Aircraft compared to drop sondes

Thermodynamic variables

	T	Logθ	h	v
H_z	0.47±0.09	0.47±0.09	0.65±0.06	0.46±0.05

L+S, 2013

14500 aircraft trajectories: $H_z = 0.57 \pm 0.01$ (Pinel, L+ S 2012)

Lidar aerosol cross sections: $H_z \approx 0.53 \pm 0.02$ (Lilley, L+S, Strawbridge, 2008)

16 Clousat orbits, radar reflectivity: $H_z = 0.56 \pm 0.04$ (L+ S, Tuck 2009)

Historical development of GCM's: $H_z \approx 5/9$ (L 2019)

$\approx 10^6$ CloudSat cloud heights, thicknesses: $H_z = 0.53 \pm 0.02$ (L 2021)

Conclusion:
The D_{el} = 2+Hz=23/9= 2.55 model is well supported by diverse data

New and old results on the divergence of moments at high Re turbulence

Theoretical prediction of multifractal processes

$$\Pr\left(\Delta v > s\right) \approx s^{-q_D} \longleftrightarrow \left\langle \Delta v^q \right\rangle \to \infty; \quad q > q_D$$

Large threshold s

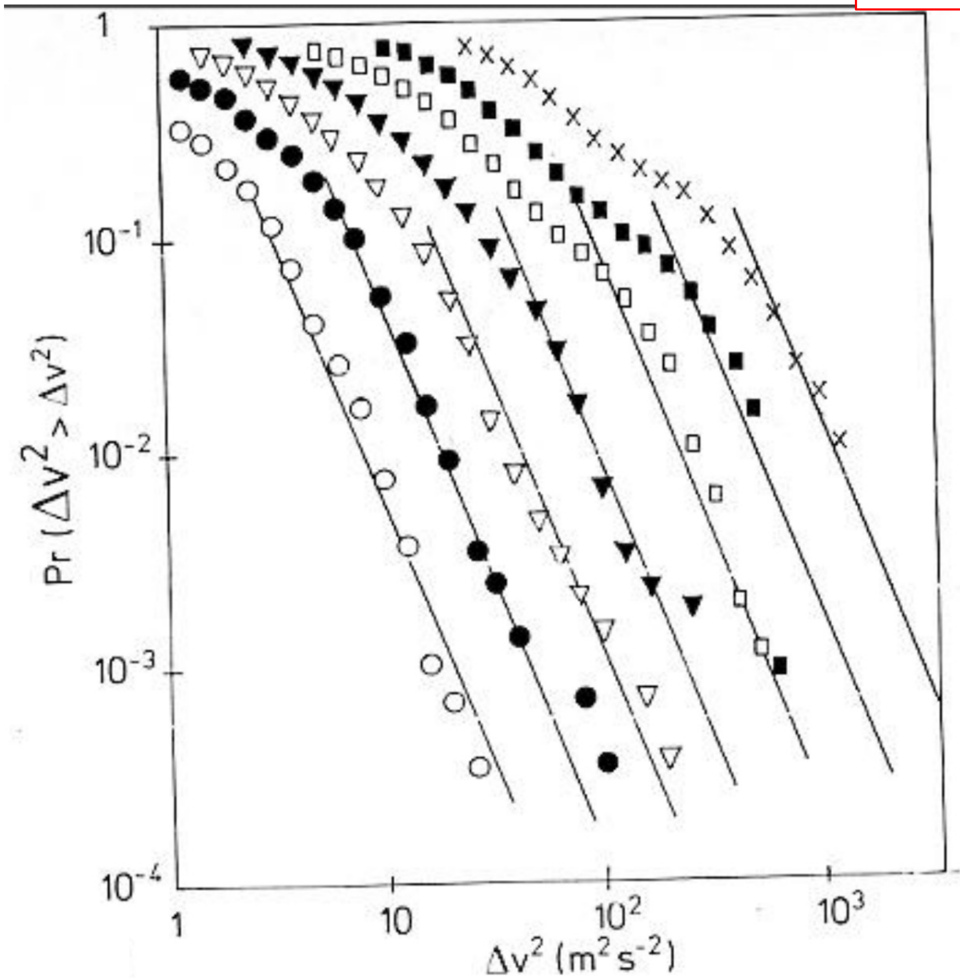
$$K\left(q_D\right) = D\left(q_D - 1\right)$$

Multifractal theory
(Mandelbrot 1974, Kahane and Peyriere 1976, S+L 1983)

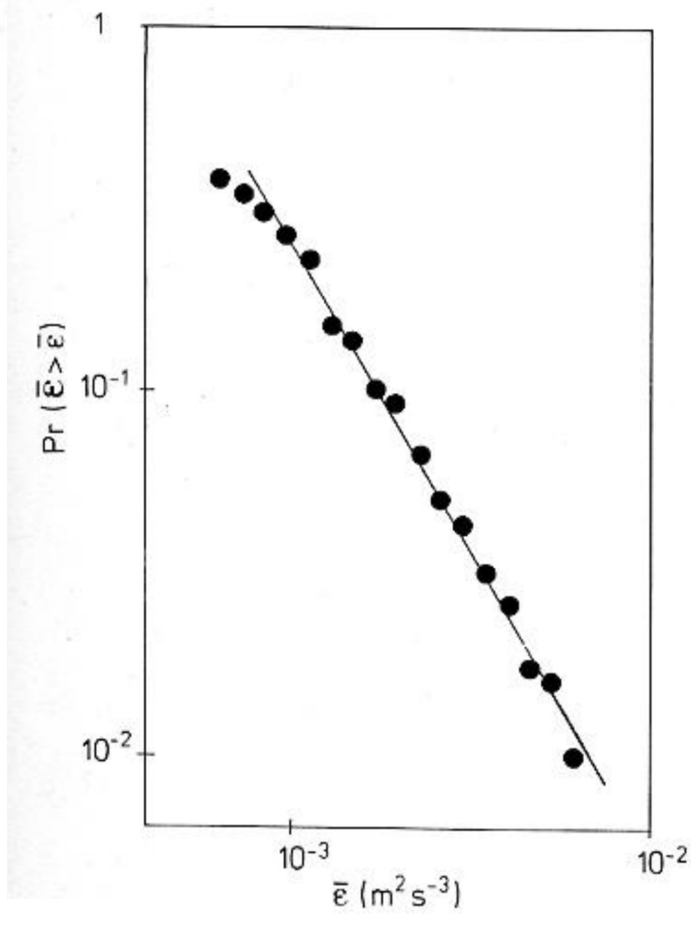
Wind (Vertical direction)

1985

Wind (Horizontal direction)



Horizontal wind in
the , vertical
From Radiosondes



Horizontal wind
(aircraft data)

$$\Pr(\Delta v > s) \approx s^{-q_{D,v}}$$

$$\Pr(\varepsilon > s) \approx s^{-q_{D,\varepsilon}}$$

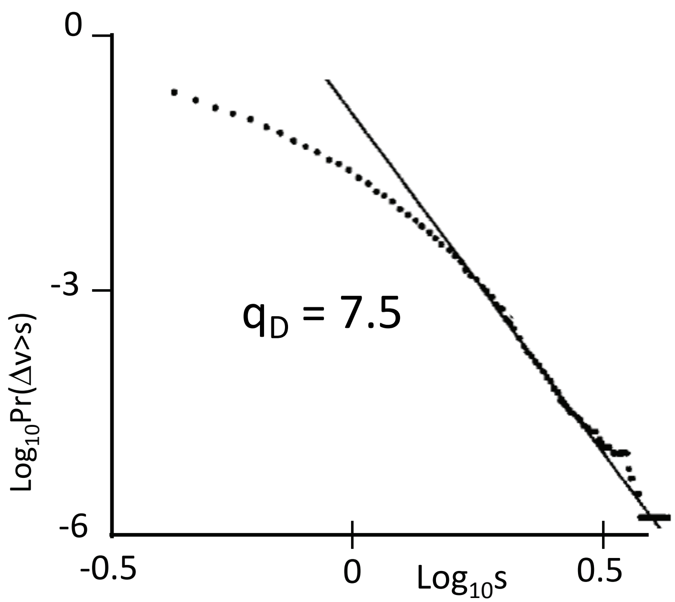
S+L 1985

$$q_{D,v} = 3q_{D,\varepsilon} \approx 5$$

$$\varepsilon = \Delta v^3 / \Delta x$$

Wind Time

1994

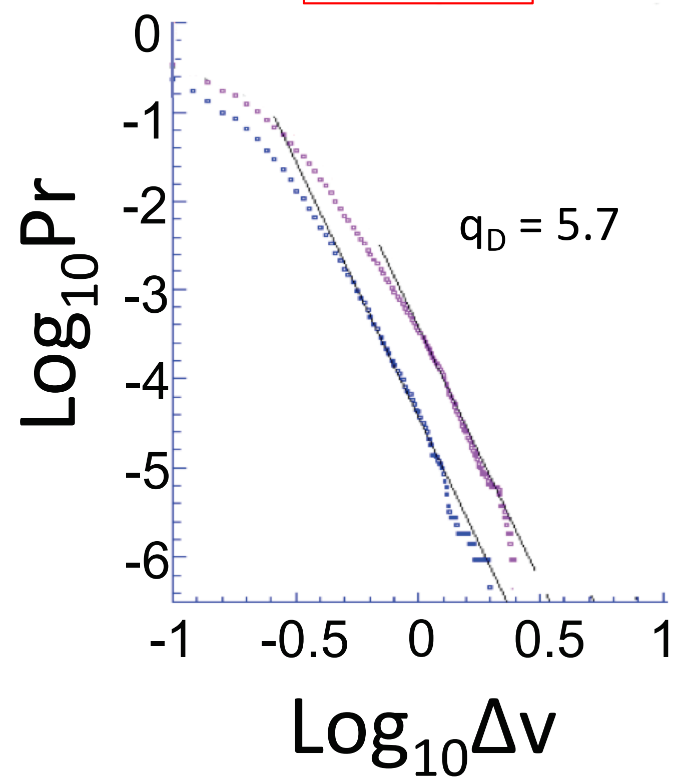


Wind fluctuations, time, 10 Hz,
(sonic anemometer)

Schmitt, S+ L, Brunet, 1994

Wind Horizontal

2007

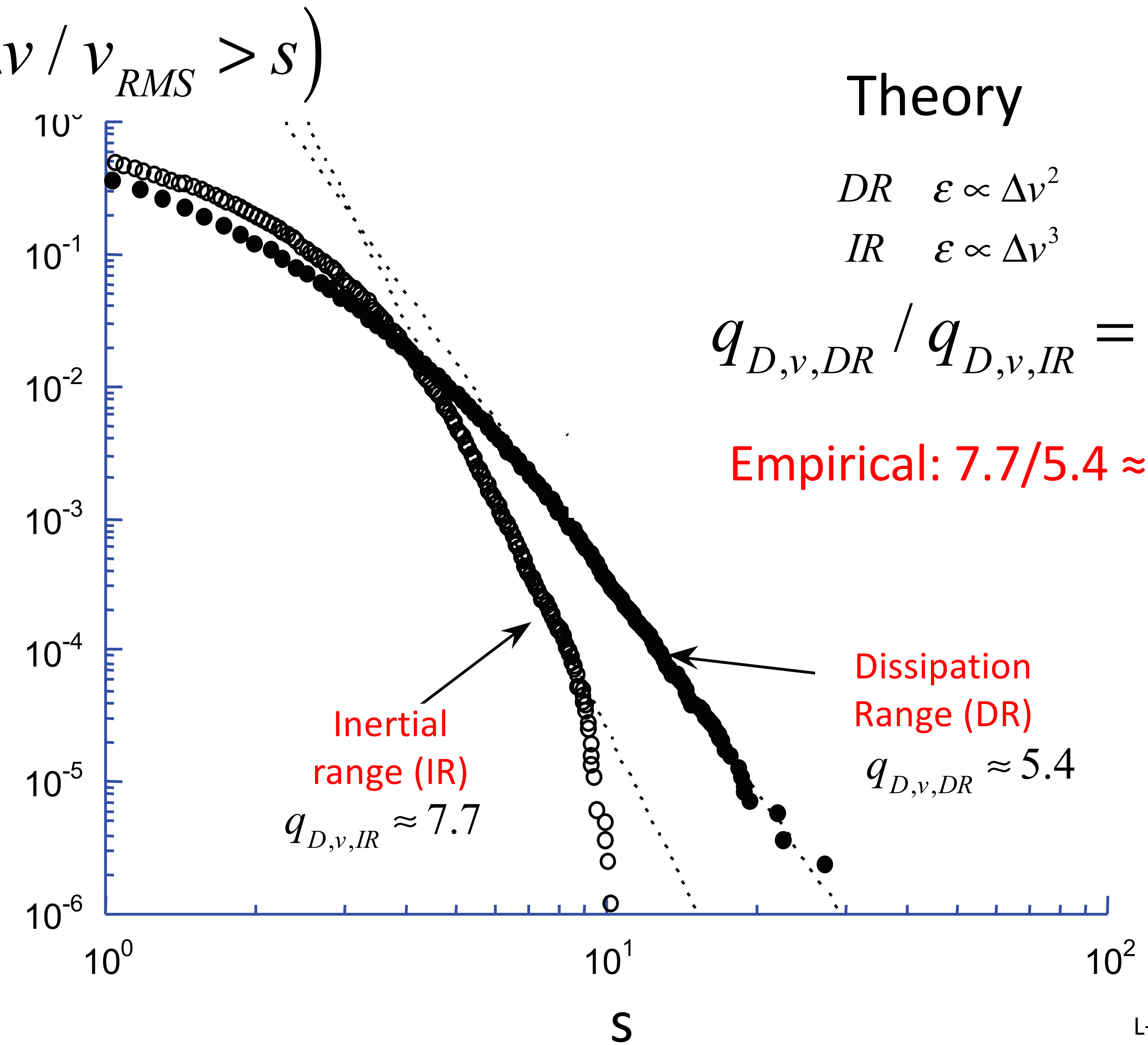


Aircraft data at 40, 80 m separation

L+S, 2007

Wind tunnel turbulence

2013



Theory

$DR \quad \varepsilon \propto \Delta v^2$

$IR \quad \varepsilon \propto \Delta v^3$

$$q_{D,v,DR} / q_{D,v,IR} = 3/2$$

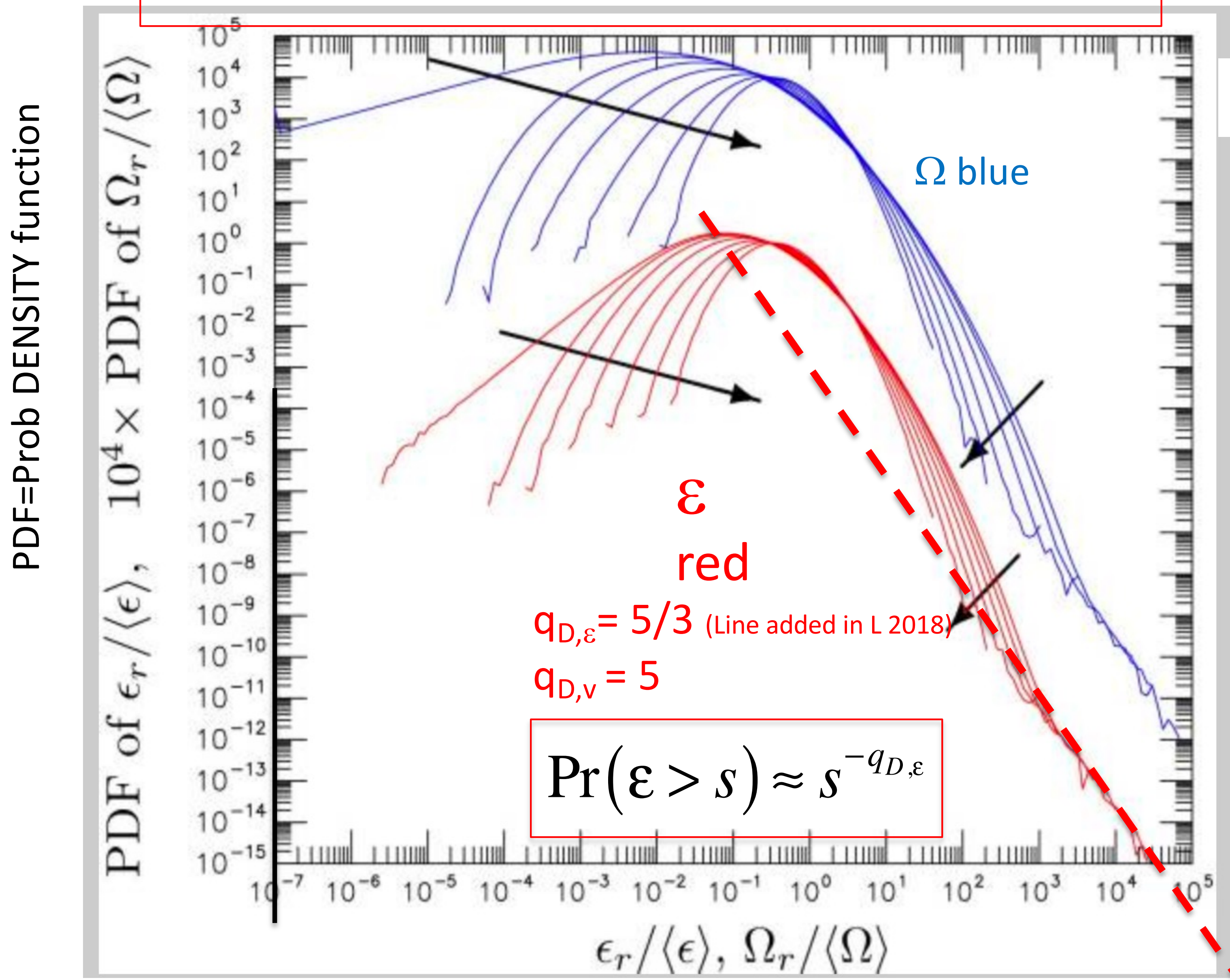
Empirical: 7.7/5.4 ≈1.43

L+S 2013

Direct Numerical Simulations

Cube: $2^{13} \times 2^{13} \times 2^{13}$

Yeunga, Zhaib, Sreenivasan 2015

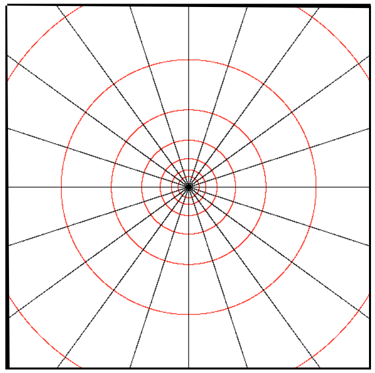


Conclusion of all studies: $q_{Dv} \approx 5$

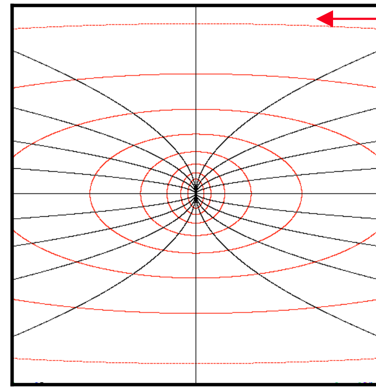
Generalized Scale Invariance

Scale functions in linear GSI (position independent)

Isotropic (self similar)



$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



Scale isolines in red $\|\underline{r}\|$=contstant

Self-affine

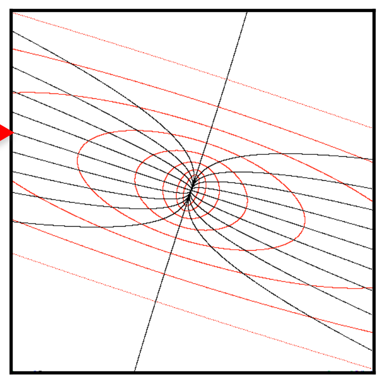
$$G = \begin{pmatrix} 1.35 & 0 \\ 0 & 0.65 \end{pmatrix}$$

$$T_\lambda = \lambda^{-G}$$

Scale functions

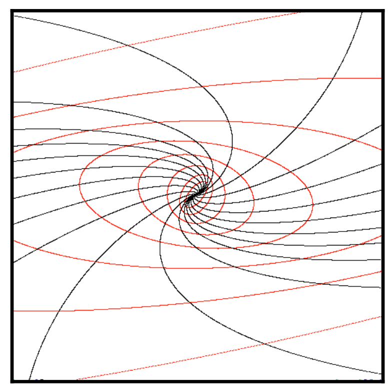
$$\|\lambda^{-G}\underline{r}\| = \lambda^{-1}\|\underline{r}\|$$

Stratification dominant (real eigenvalues)



$$G = \begin{pmatrix} 1.35 & 0.25 \\ 0.25 & 0.65 \end{pmatrix}$$

Rotation dominant (complex eigenvalues)

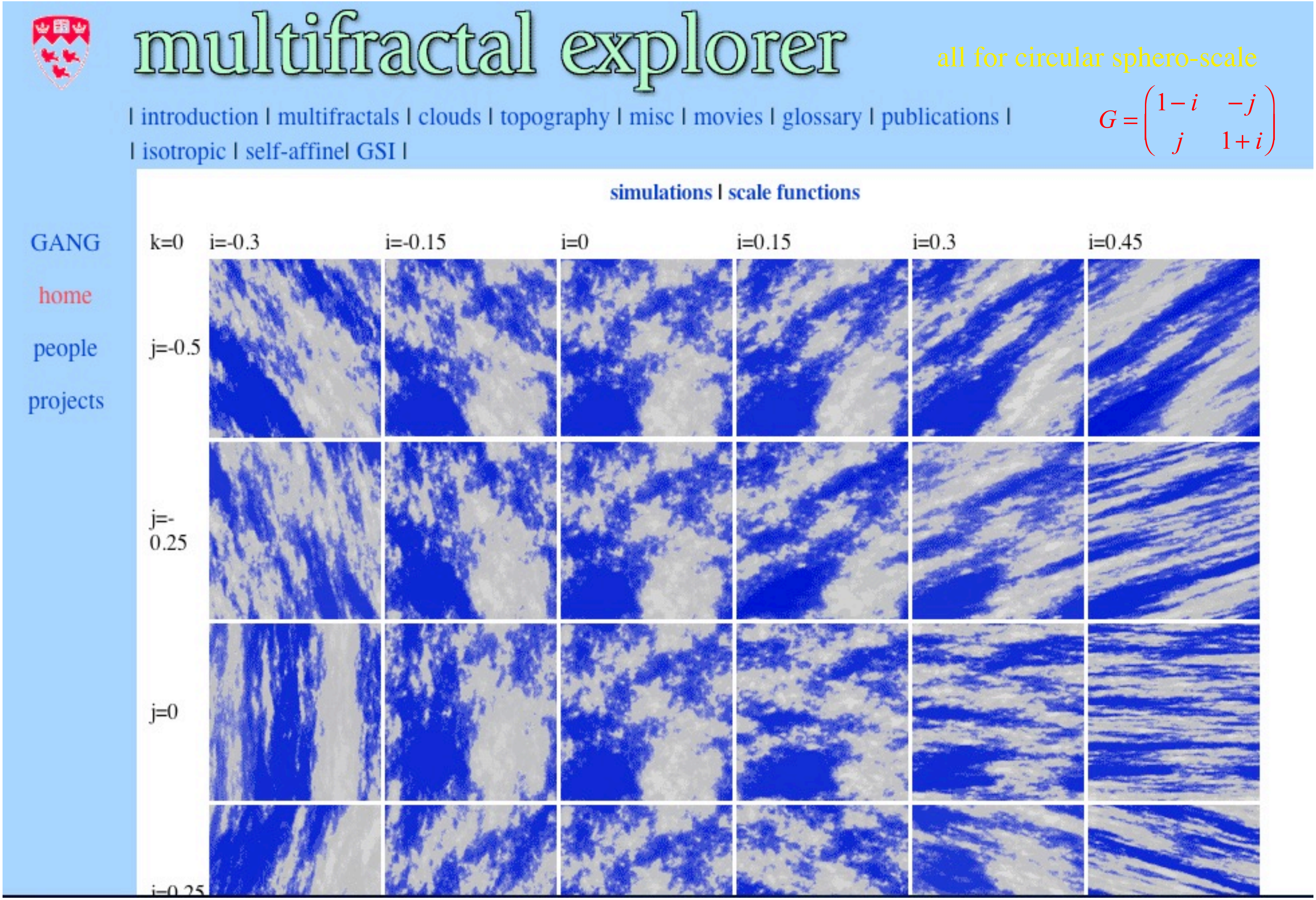


$$G = \begin{pmatrix} 1.35 & -0.45 \\ 0.85 & 0.65 \end{pmatrix}$$

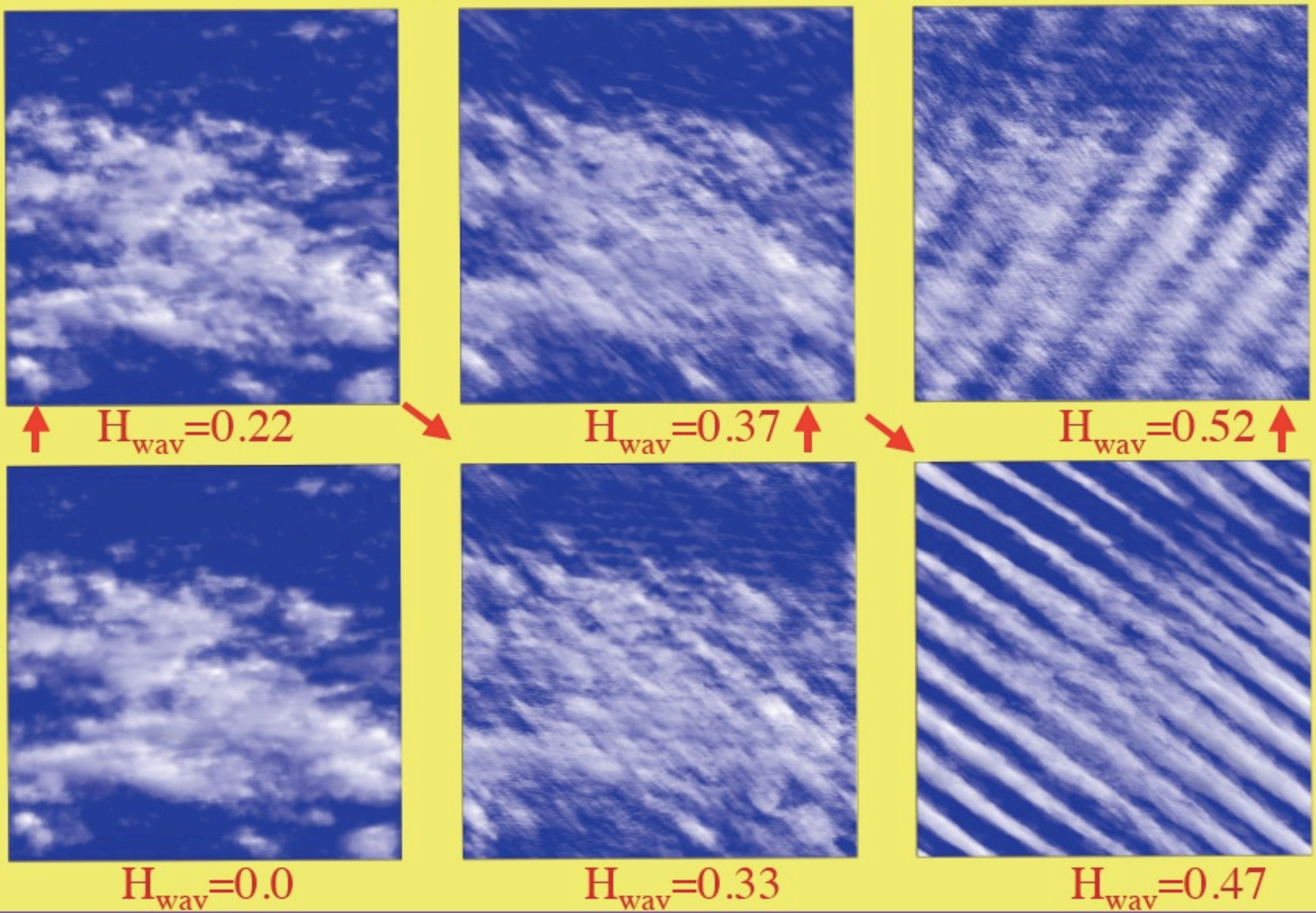
S+L 1985

Changing G

http://www.physics.mcgill.ca/~gang/multifrac/index.htm



Cascades from localized to increasingly unlocalized structures: $H_{wav} = 1/3 - H_{tur}$



Conclusions

1) Temporal Scaling defines the five main dynamical dynamical regimes: weather, macroweather, climate, macroclimate, megaclimate

2) Scaling allows for a quantitative understanding of the atmosphere whereas usual scale bound approaches are qualitative and do not agree with the data (error 10^{15}).

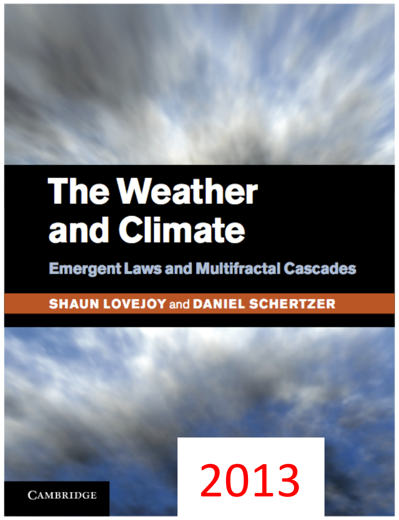
3) Origin of scaling is the (anisotropic) scaling of the governing equations (including of wind and boundary conditions)

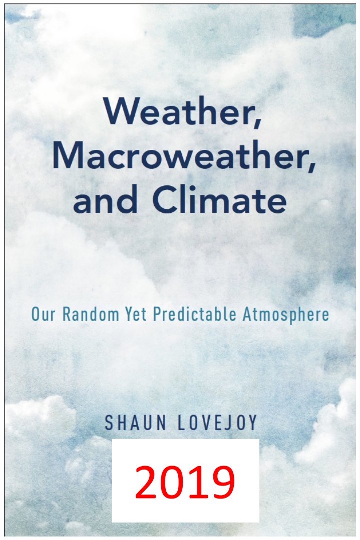
4) Multifractals are the generic scaling process: atmospheric fields show excellent cascades structures up to planetary scales (horizontal) and ≈ 10km (vertical).

5) This is possible due to anisotropic scaling (Generalized Scale Invariance)

6) Extreme events/ divergence of moments as theoretically predicted (order ≈5 for velocity)

7) Realistic space-time simulations are possible





References:
http://www.physics.mcgill.ca/~gang/reference.list.htm