
Turbulence Effecting Windturbines

modern wind turbines

power from wind

$$E_{wind} = \frac{1}{2} m u^2$$

$$P_{wind} = \dot{E}_{wind} \quad \dot{m} = \rho \dot{V} \\ = \frac{1}{2} \dot{m} u^2 \quad = \rho A \cdot u$$

$$P_{wind} = \frac{1}{2} \rho A u^3$$

A cross section, ρ air density



modern wind turbines

power from wind

$$E_{wind} = \frac{1}{2} m u^2$$

$$\begin{aligned} P_{wind} &= \dot{E}_{wind} & \dot{m} &= \rho \dot{V} \\ &= \frac{1}{2} \dot{m} u^2 & &= \rho A \cdot u \end{aligned}$$

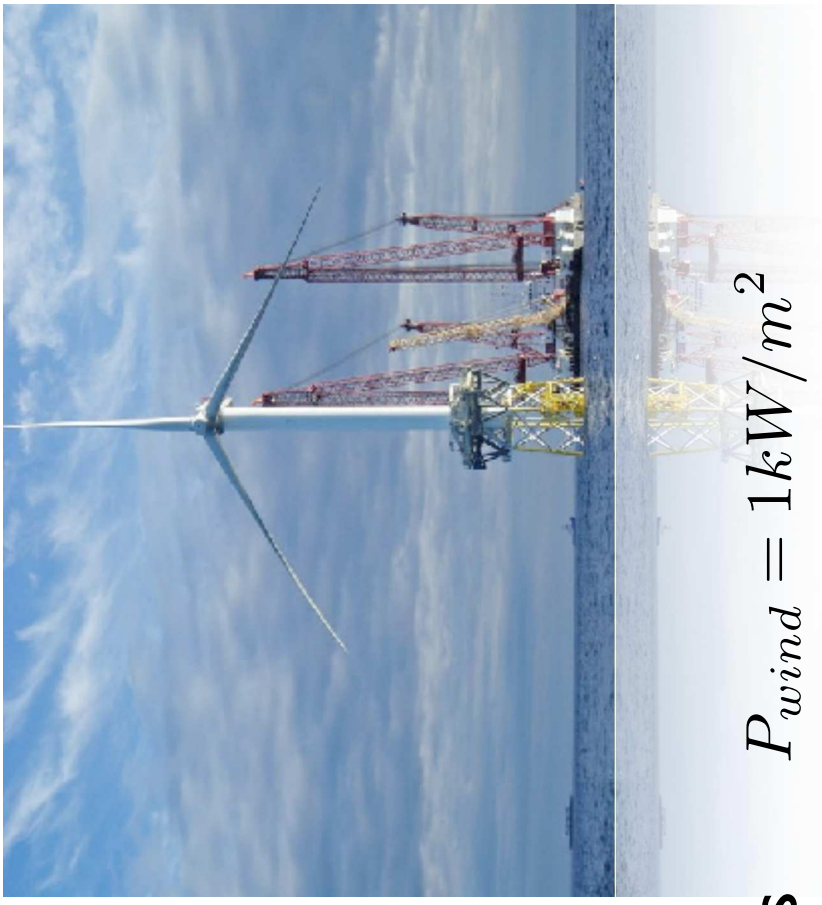
$$P_{wind} = \frac{1}{2} \rho A u^3 \quad \text{for } u = 12 \text{ m/s}$$

$$P_{wind} = 1 \text{ kW/m}^2$$

WEC

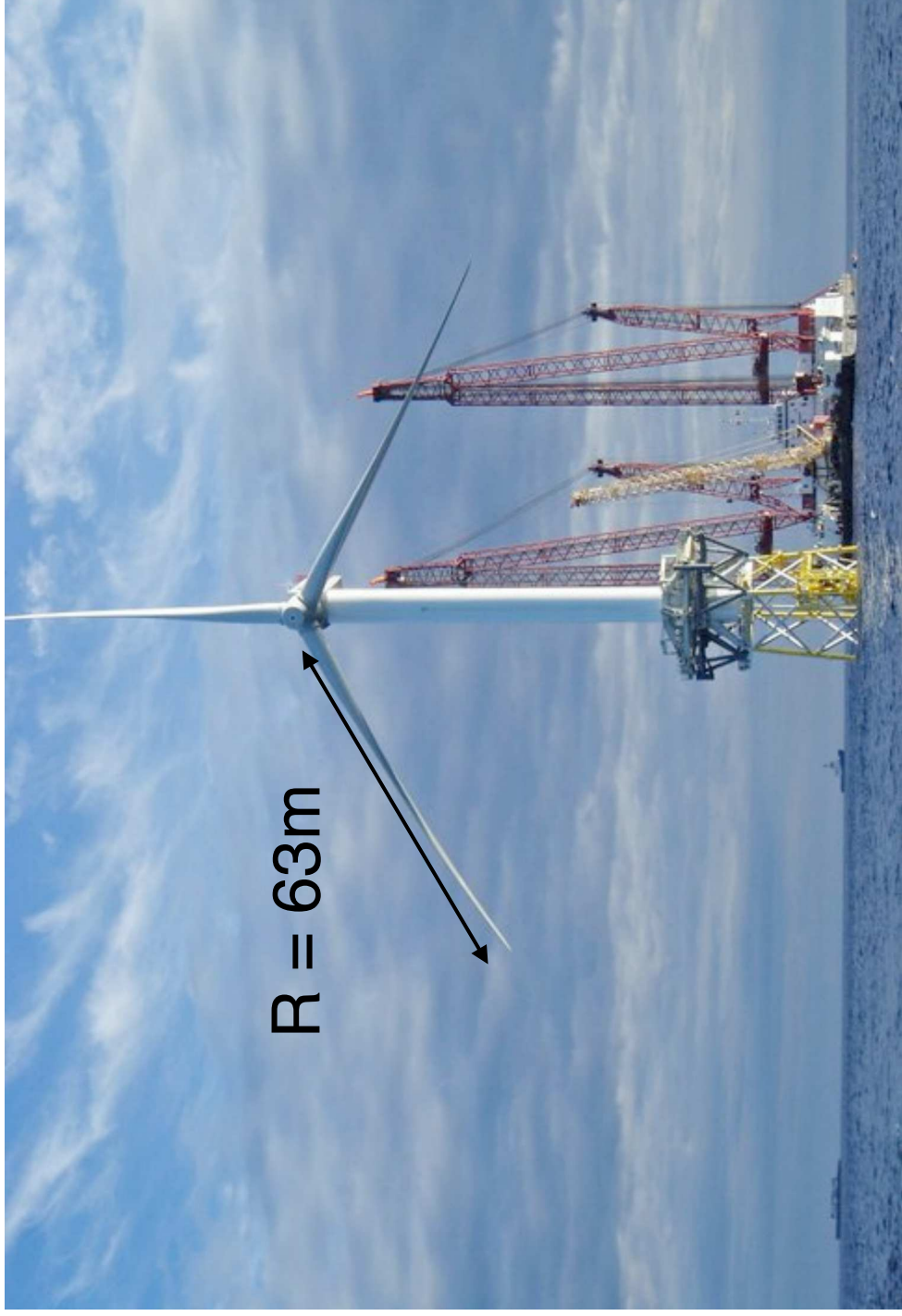
$$P_{WEC} = c_P \frac{1}{2} \rho A u^3$$

$$c_P \leq 0.59 \quad \text{Betz- Joukowski limit}$$



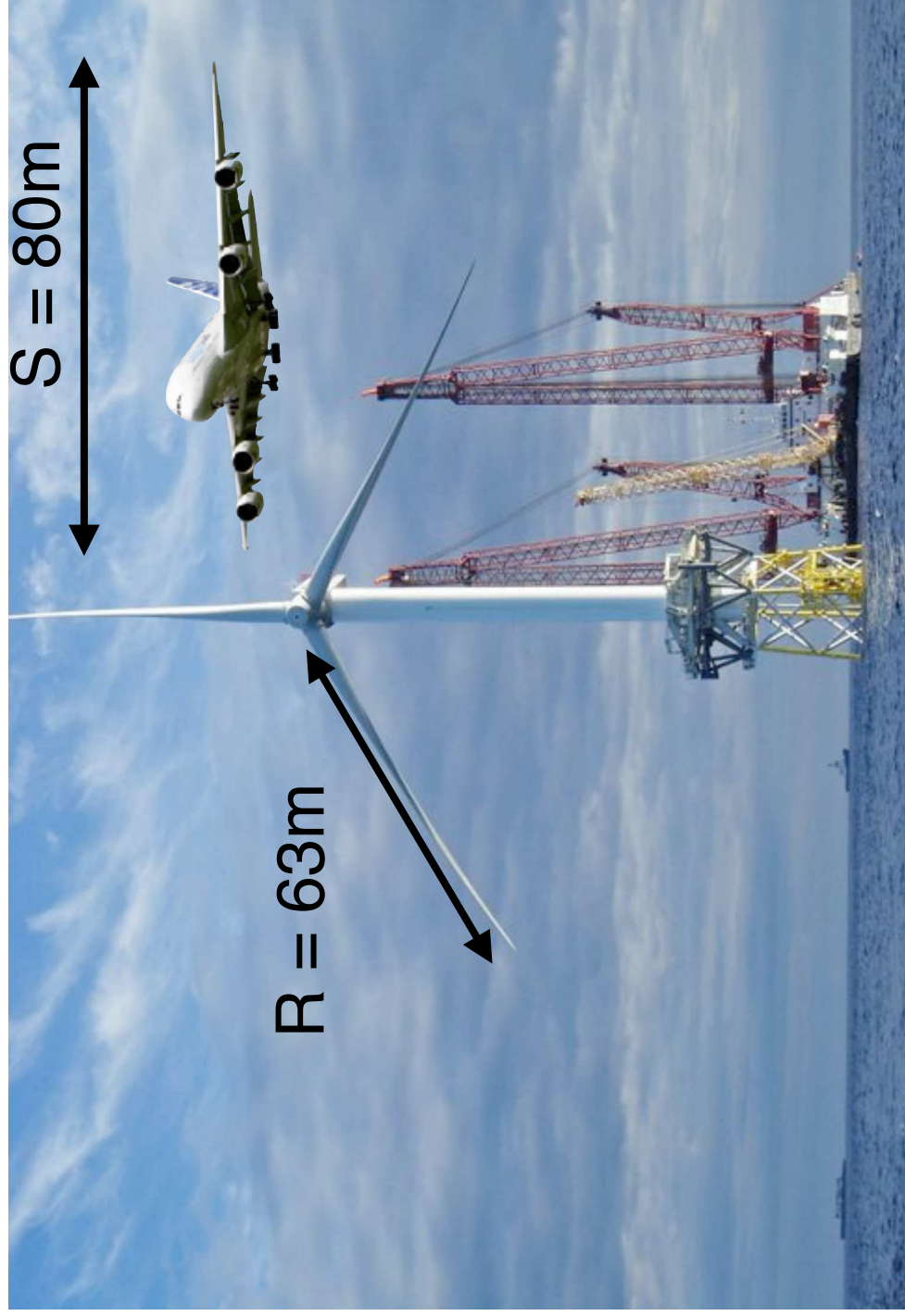
modern wind turbines

size : area = 12469 m² → 6MW - here 5MW turbine



R = 63m

modern wind turbines size



modern wind turbines

power from wind

$$E_{wind} = \frac{1}{2} m u^2$$

$$P_{wind} = \dot{E}_{wind} \quad \dot{m} = \rho \dot{V}$$

$$= \frac{1}{2} \dot{m} u^2 = \rho A \cdot u$$

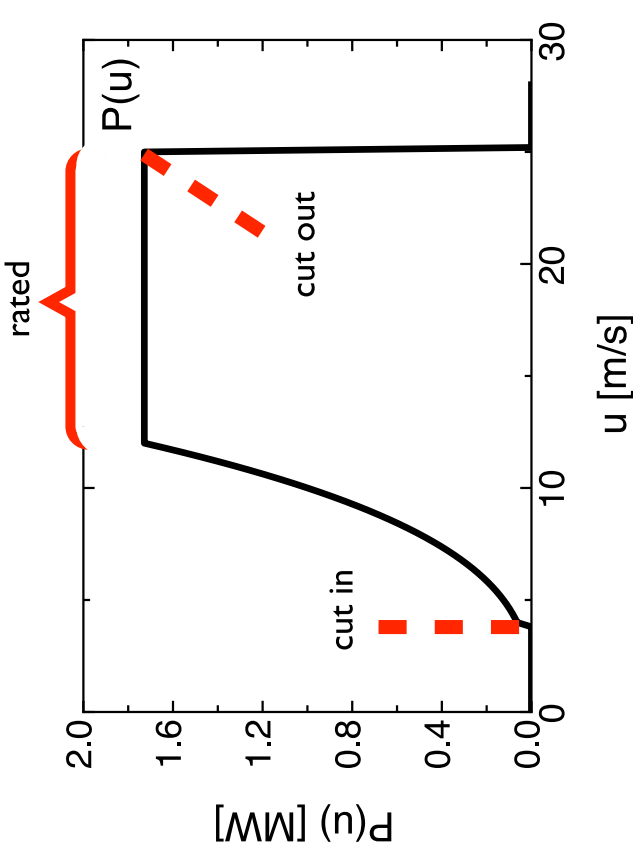
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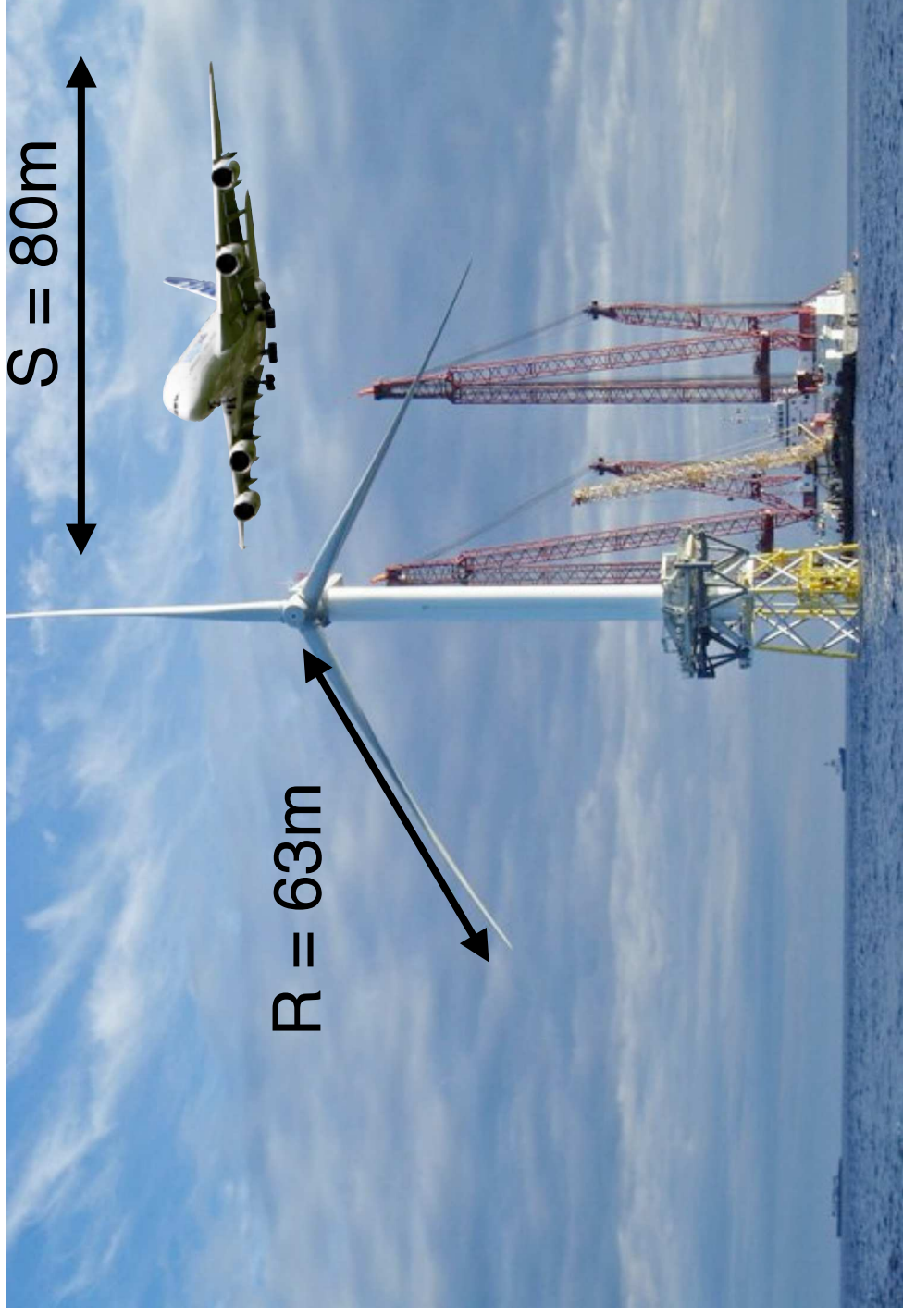
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power characteristic



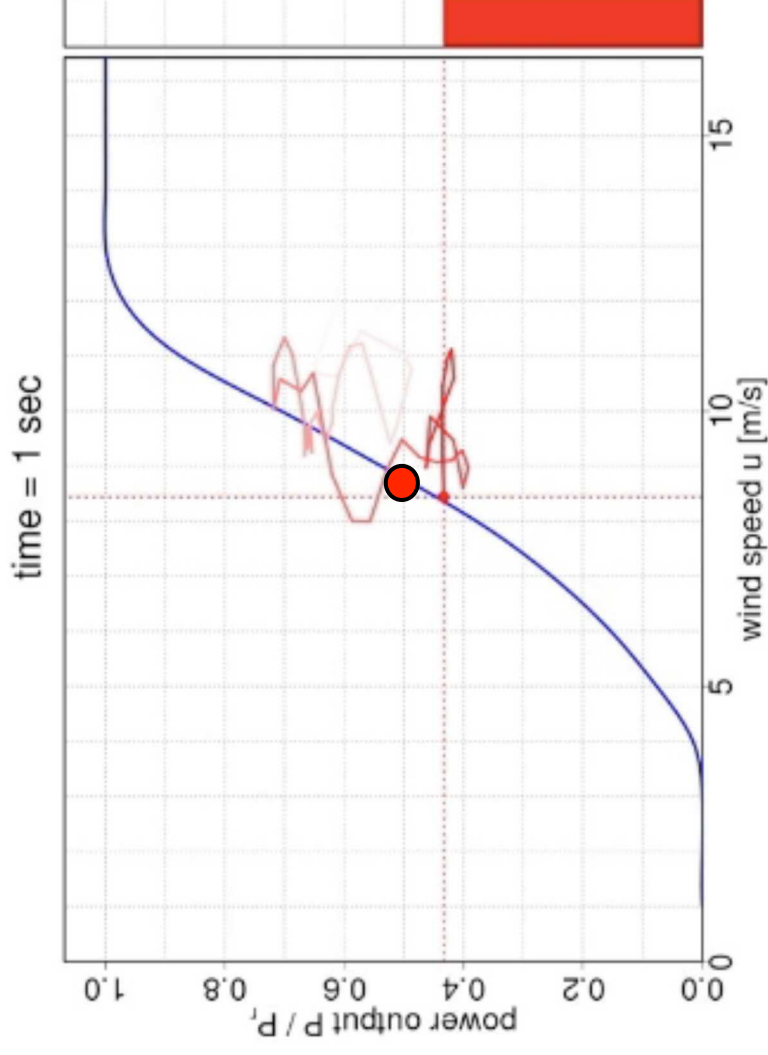
modern wind turbines how dynamic is the conversion process?



Part 2

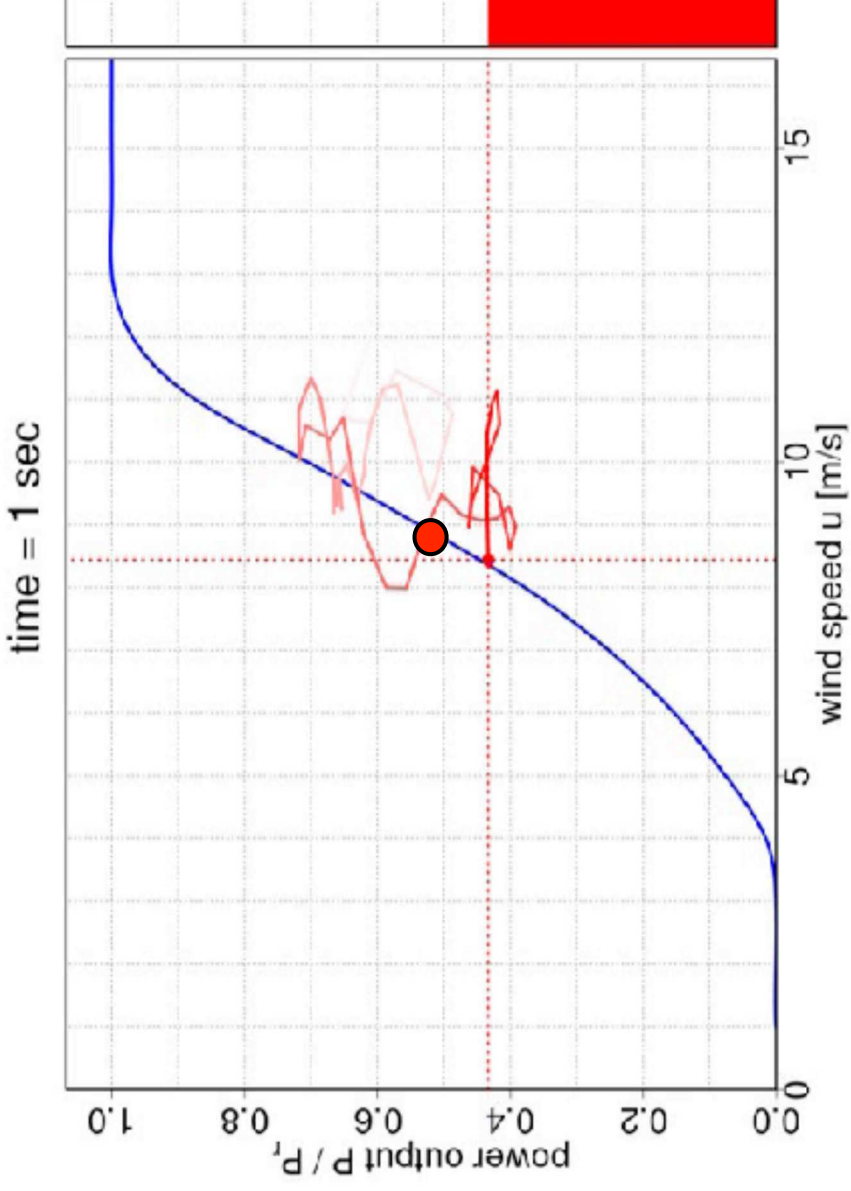
motivation : dynamics of power conversion

$$P_{WT} = \frac{1}{2} c_p(\lambda) \rho u_{wind}^3 \cdot A$$



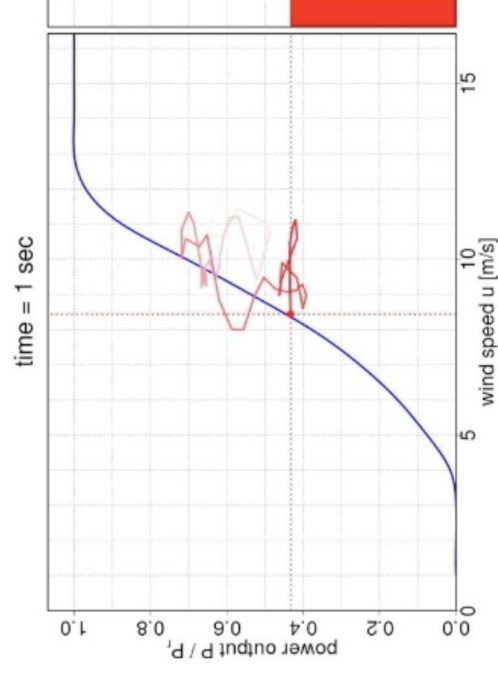
<http://phys.org/news/2013-04-turbines-great-turbulence-consequences-grid.html>

Part 2 motivation : dynamics of power conversion



<http://phys.org/news/2013-04-turbines-great-turbulence-consequences-grid.html>

**highly fluctuating power output,
what is the reason?
does turbulence of wind pay a role?**



Turbulence one of 7 millenium problems



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EXISTENCE AND SMOOTHNESS OF THE NAVIER-STOKES EQUATION

CHARLES L. FEFFERMAN

The *Navier-Stokes* equations are then given by

$$(1) \quad \frac{\partial}{\partial t} u_i + \sum_{j=1}^n u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x, t) \quad (x \in \mathbb{R}^n, t \geq 0),$$

$$(2) \quad \operatorname{div} u = \sum_{i=1}^n \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0)$$

$$(11) \quad p, u \in C^\infty(\mathbb{R}^n \times [0, \infty)).$$

A fundamental problem in analysis is to decide whether such smooth, physically reasonable solutions exist for the Navier-Stokes equations. To give reasonable leeway to solvers while retaining the heart of the problem, we ask for a proof of one of the following four statements.

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$$\frac{\partial}{\partial x}u(x) = \lim_{r \rightarrow 0} \frac{u(x+r) - u(x)}{r}$$

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velocity increment

$$\frac{\partial}{\partial x} u(x) = \lim_{r \rightarrow 0} \underbrace{u(x+r) - u(x)}_{u_r} \frac{1}{r}$$

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u_r → velocity increment

$$\begin{aligned}\frac{\partial}{\partial x}u(x) &= \lim_{r \rightarrow 0} \frac{\overbrace{u(x+r) - u(x)}^{u_r}}{r} \\ &= \lim_{r \rightarrow 0} \frac{u_r}{r}\end{aligned}$$

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u_r → velocity increment

$$\frac{\partial}{\partial x} u(x) = \lim_{r \rightarrow 0} \frac{\overbrace{u(x+r) - u(x)}^{u_r}}{r}$$

have to understand

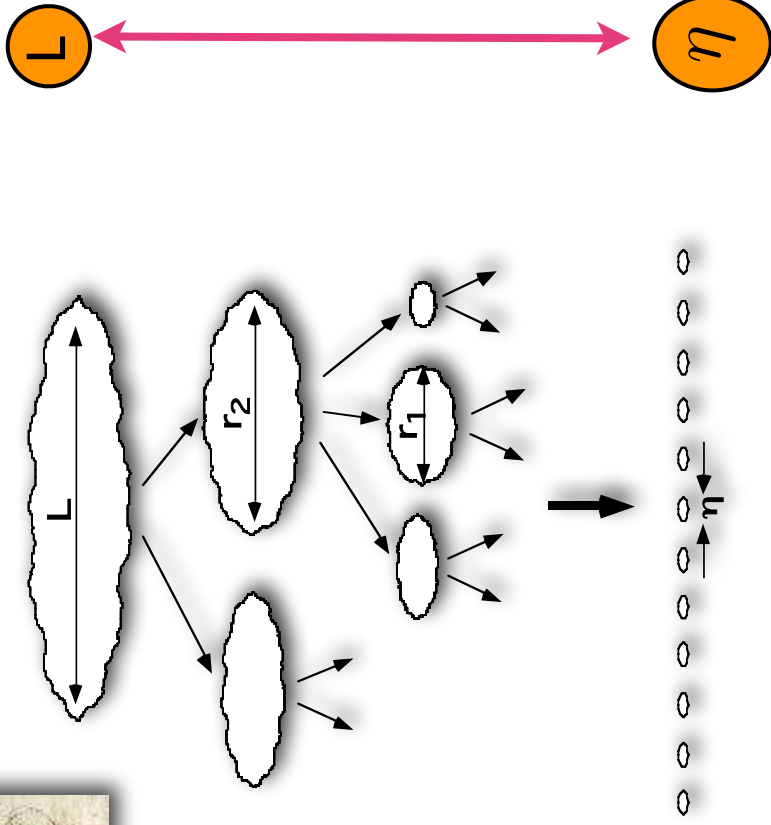
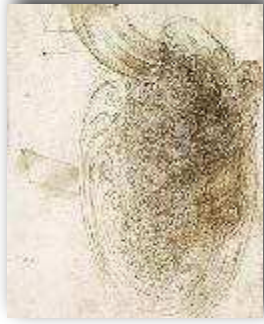
$$= \lim_{r \rightarrow 0} \frac{u_r}{r}$$

$$\lim_{r \rightarrow 0} u_r$$

homogeneous isotropic turbulence -- hit

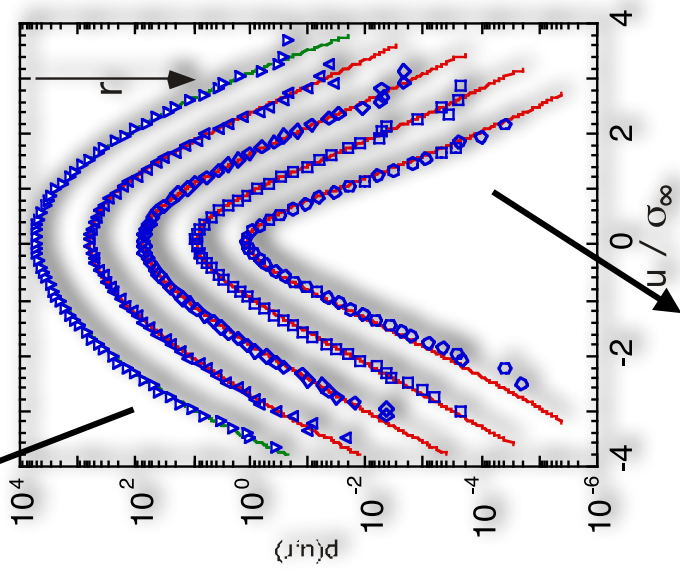
▼ r - depend of velocity increments: $u_r = u(x+r) - u(x)$

- cascade and **statistics of increments**



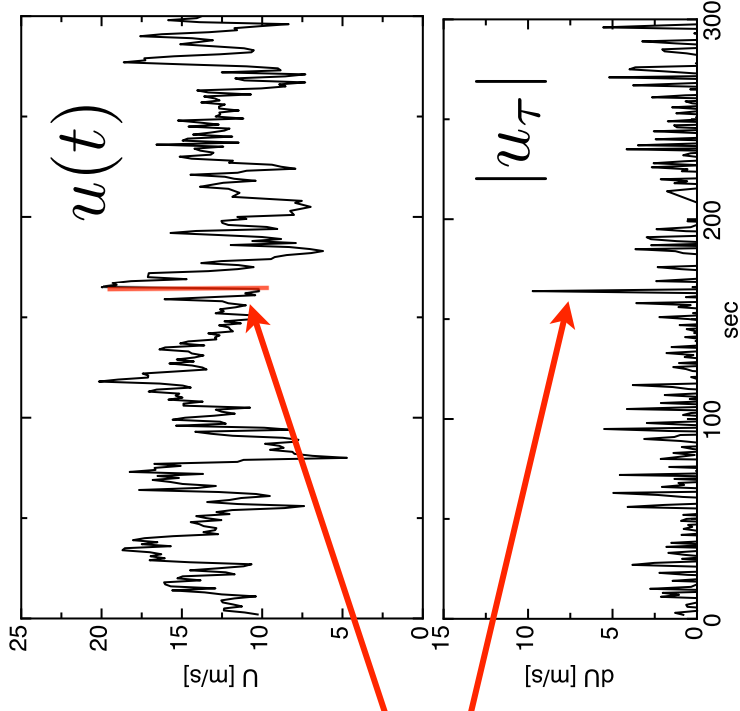
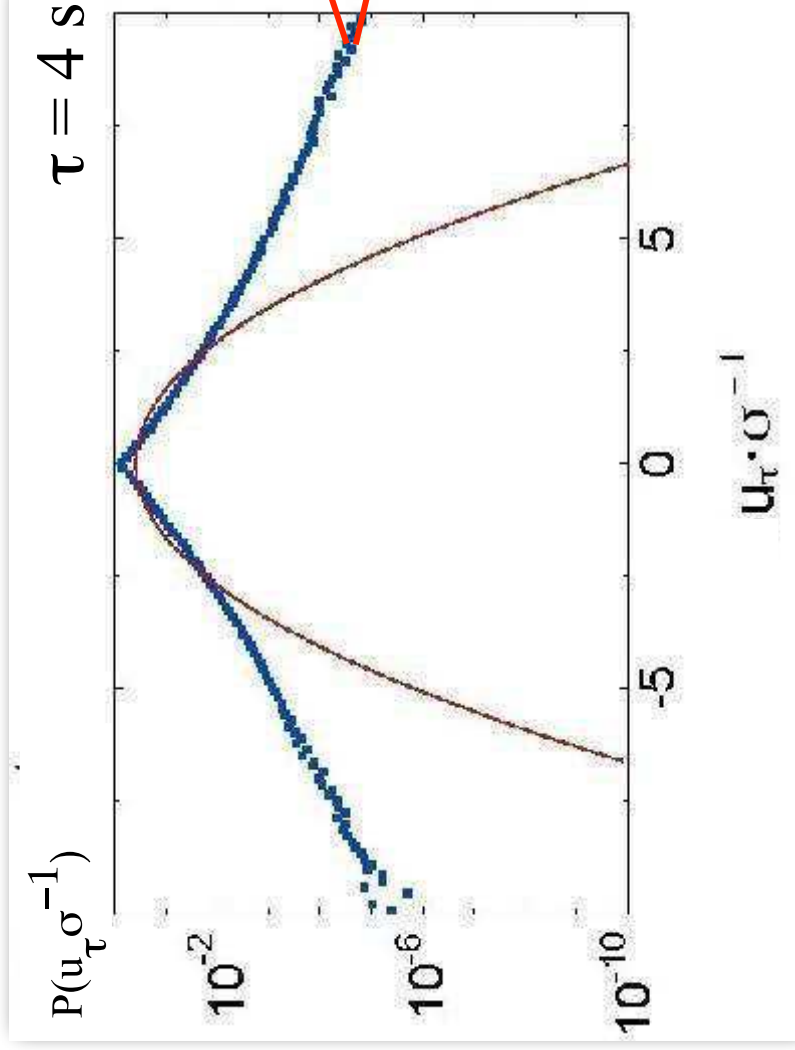
A.N. Kolmogorov

Gaussian



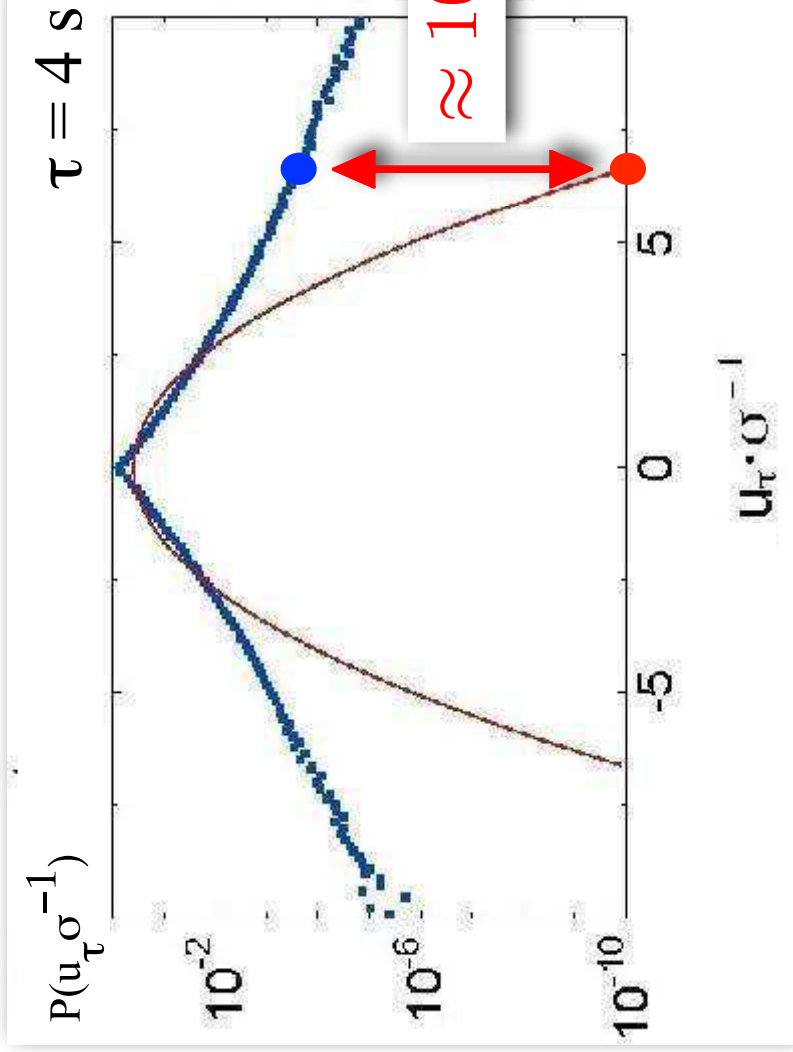
intermittent

statistics of turbulent wind: $u_\tau = u(t + \tau) - u(t)$



Boundary-Layer Meteorology **108** (2003)

statistics of gusts



Boundary-Layer Meteorology **108** (2003)

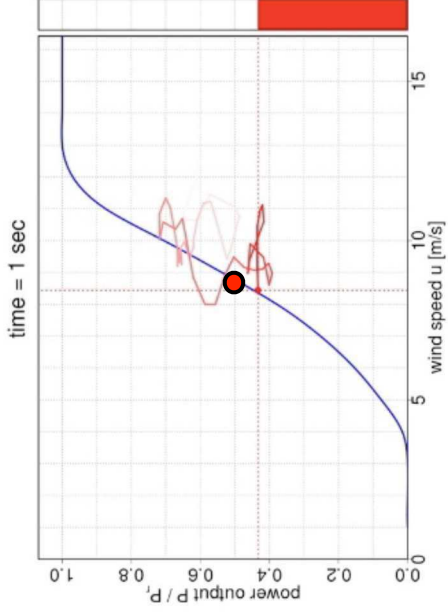
$Prob(u_\tau > 6\sigma) \approx 10^{-4}$
 1/day

$Prob(u_\tau > 6\sigma) \approx 10^{-10}$

1/3000 years

what is the power statistics?

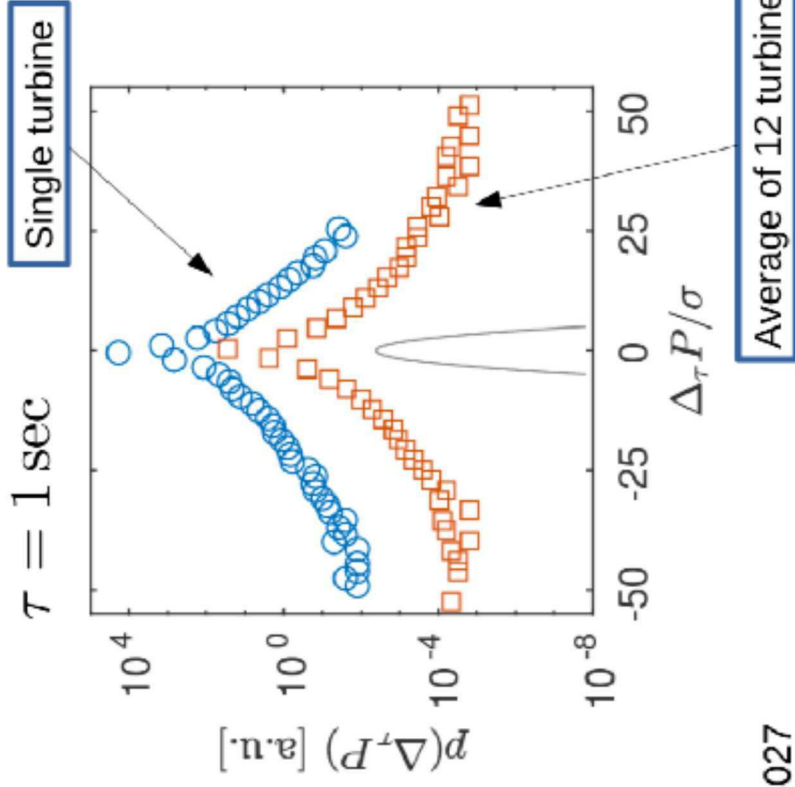
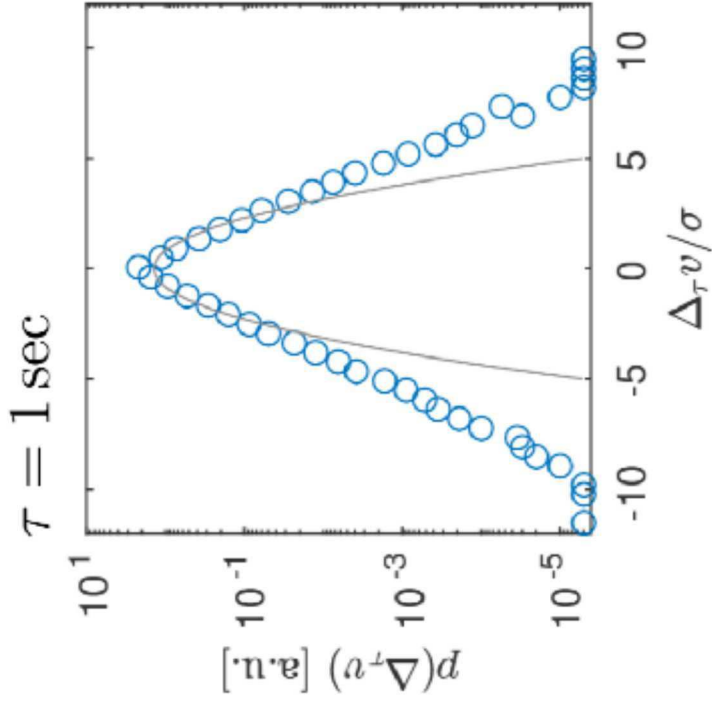
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power output of wind energy

turbulent power dynamics

$$p(P_\tau)$$



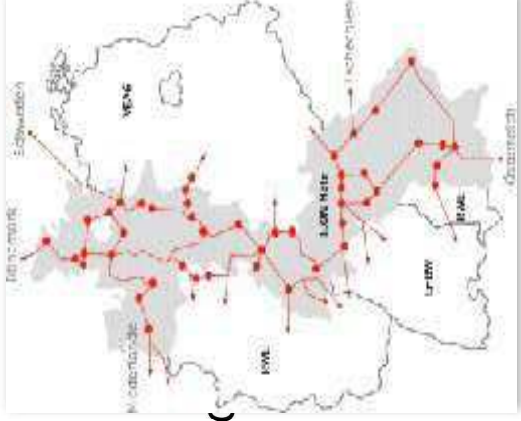
M. Anvari et al., *New J. Phys.* **18** (2016) 063027

handling high frequency

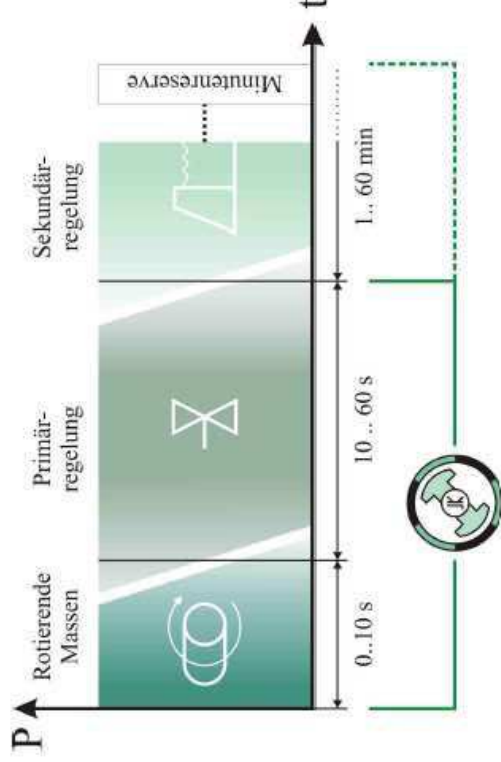


classical power generation →

→ local renewable



short time grid frequency
 stabilisation by rotating mass
 of synchronous generators —
 grid frequency changes in the
 range of mHz
www.netzfrequenzmessung.de

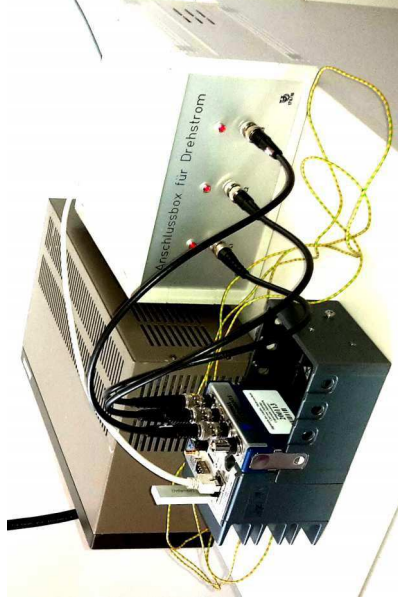
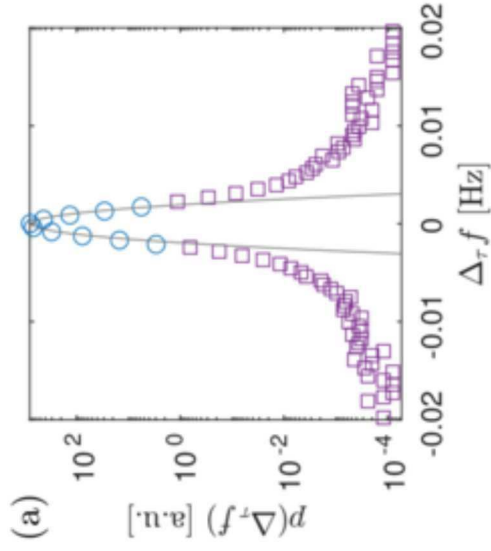
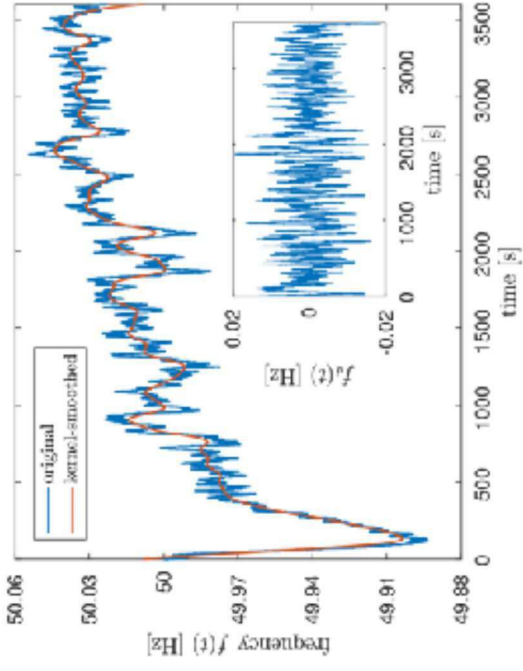


- intermittency in grid - measurements

analysis of the grid frequency
measurement at the plug connector

- frequency increments

$$\tau < 1 \text{ sec}$$

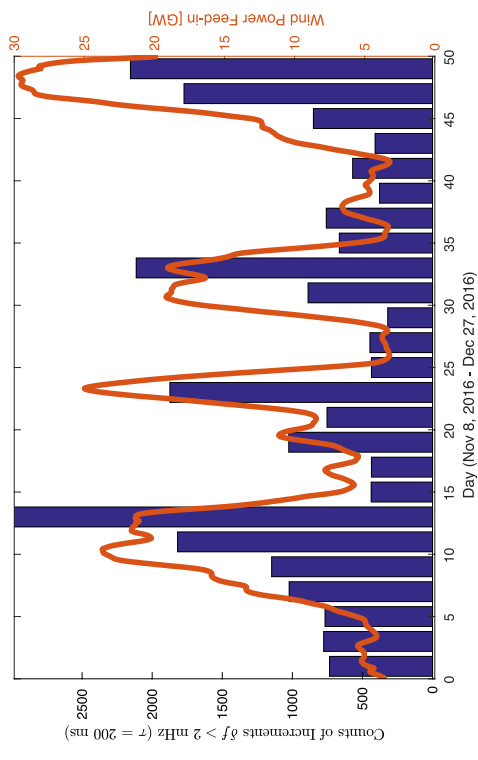
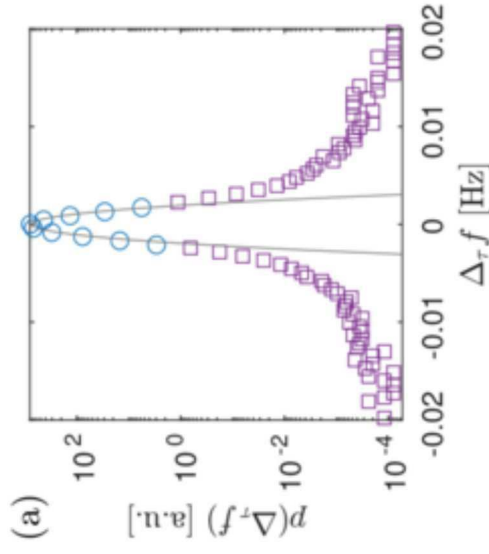
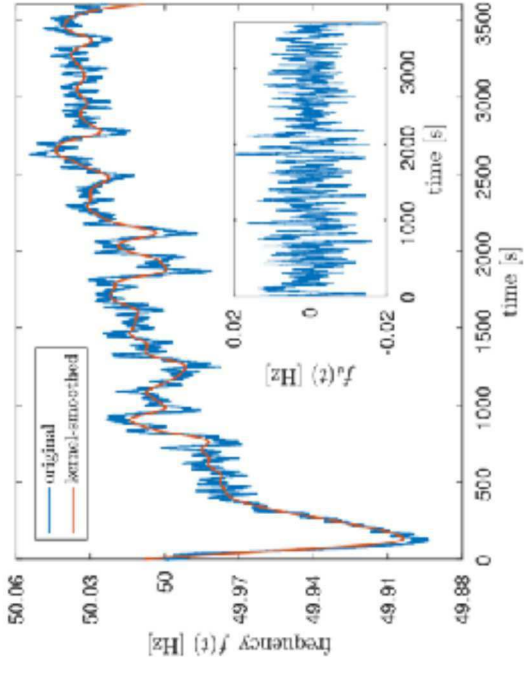


H. Hähne, et al. Europhysics Letters 121, 30001 (2018)

- intermittency in grid

analysis of the grid frequency

— clear fingerprint of renewable energies



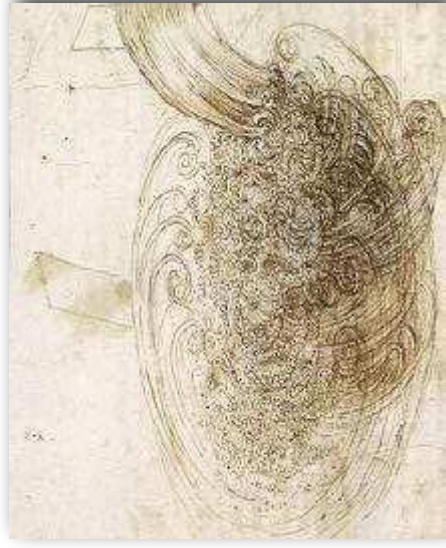
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wind energy is driven by turbulence

Inflow is energy resource



claim - need to understand turbulence
and other details of the wind



Oldenburg

- free field measurements. Lidar
- CFD - turb inflow LES
- wind tunnel, active gid

